

ASTROGATION

Rulebook and Resources

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Part I

Core References

1. Introduction

The Astrogating Game for Mental Masturbation

You are a mission planner. You have a budget, a payload, and the laws of physics. Everything between Earth's surface and the edge of the solar system is fair game. Design rockets. Plot trajectories. Calculate burns. Launch missions. The universe doesn't care about your intentions—only your math.

DESIGN PHILOSOPHY

- **Ultra-concise core manual.** If it can be derived from earlier content without heroic effort, it's not here. The physics handbook gives you the equations; you do the algebra.
- **Maximum freedom, grounded in reality.** Nothing is forbidden except the impossible. Want to launch a crewed mission to Proxima Centauri using only chemical rockets? You can try. The numbers will tell you why nobody does.
- **The universe expands through play.** Propose what might exist—platinum asteroids, alien microbes, breakthrough propulsion. Argue your case. Roll the dice. What you discover becomes real. The known is your starting point, not your ceiling.
- **Two players, both rule-makers.** No GM. Both players propose, challenge, and resolve. The other player is your check against wishful thinking. Evidence beats assertion.
- **One tool required: a graphing calculator.** No spreadsheets. No simulation software. No LLM. Everything you need for orbital mechanics, rocket design, and mission planning fits in these pages and your hands.

STRUCTURE

- **Gameplay** — Companies, funding, time and cash, reputation, the loop.
- **The Shop** — Everything you can buy or build, and what it costs.
- **Making Stuff** — Labs and factories: spin up, research, manufacture.
- **Personnel** — Staff (founder included): postings, floor, attention, flight crew, leave, burnout.
- **Physics Handbook** — Equations for trajectories, rockets, and orbital mechanics.
- **Astrodata** — Positions, compositions, and constants for the solar system.
- **Chance** — The universal mechanic: when things happen, what fails, what you discover.
- **Odds** — What breaks and how. Failure modes and consequences.
- **Exercises** — Separate addendums: full rocket builds, trajectory plans, mission walkthroughs. Core chapters keep one-line examples only.

FIRST SESSION

Start with Gameplay. Found a company, get funding. Pick a first mission from the Shop — a rideshare deployment or a simple satellite. Use Physics Handbook only when you need a number. Roll for outcomes per Chance. Look up failures in Odds. Learn by doing.

Second draft. Everything is subject to revision, including the laws of physics if they prove insufficiently entertaining.

2. Gameplay

2.1 STARTING THE GAME

Each player founds a company. Investors believe you're worth something—now prove them right.

Seed valuation: Agree on a baseline valuation with the other player. \$1B minimum recommended. This is speculative—you're a famous founder, a hot trend, a compelling pitch. You haven't proven anything yet.

Seed round: Decide how much equity to sell. Then roll for market sentiment.

Roll until you fail: Roll d100. If ≥ 50 , count it and roll again. If < 50 , stop. Multiplier = 1 + successes.

More capital = more runway. More dilution = less upside if you win.

Example: Baseline \$1B. You commit to selling 20%. Roll 72 (success), 55 (success), 31 (stop) → 2 successes → 3× multiplier → valuation \$3B. You get \$600M. Now prove it was deserved.

2.2 FUNDRAISING

To raise more capital during play, pitch investors based on what you've built.

Argue your baseline valuation using whatever benchmarks fit:

- **Revenue multiple** — Revenue × 10–50 (higher for growth, moat, TAM)
- **Profit multiple** — Profit × 15–100 (higher if scaling)
- **Comparables** — “Company X trades at Y; I’m similar because...”
- **Asset value** — Replacement cost of what you’ve built (floor)

The other player may challenge. Evidence beats assertion.

Raise capital: Decide how much equity to sell. Then roll for market sentiment (roll until < 50 , multiplier = 1 + successes).

Scientific funding: If the mission has scientific merit, argue a grant (NSF-like) with the other player — amount, milestones, and what counts as delivery.

Richer instruments (debt, procurement, public markets) are optional; negotiate from real-world analogues when you want them.

2.3 TIME & CASH

Track **calendar date** and **cash**. When time advances (procurement lead time, facility spin-up, coasts, waits for windows), pay ongoing costs for that span — facility ops or idle, Ground on ops, ground contact once stood up, crew, and any other \$/year lines you have open.

Example: Three months waiting on parts → no ops contact yet; stand contact up 1 month before launch (or use a LEO short campaign), assign Ground to ops, then pay through the window (Shop: Operations).

2.4 GROUNDING

- **Games begin today.** The current date. All technology, markets, and positions reflect the real world right now. Use the Shop for prices, Astrodata for positions.
- **Use what exists.** You may buy any product and sell to any market that currently exists. New markets emerge only from player actions.
- **Propose, then commit.** Before commitment, argue freely—challenge assumptions, demand derivations, dispute physics. At commitment, parameters lock. After commitment, results are canonical. No retroactive objections.

2.5 MISSIONS

A **mission** is a discrete operation with committed resources and defined success criteria. One launch with multiple payloads = one mission. Separate launches = separate missions. Lock success criteria at commit (what “done” means).

Outcomes (reputation = Shop table — mission count sets your cap; failures drop you below it):

- **Success** — Objectives achieved. Restore one reputation level (up to cap).
- **Partial failure** — Some objectives achieved ($\geq 50\%$), or hardware damaged but mission completed. -1 level.
- **Major failure** — Objectives not achieved, or vehicle/crew loss. -2 levels.

2.6 THE LOOP

1. **Set a goal** — Orbit a satellite, build a business, reach another star
2. **Design your hardware** — From the Shop (typical bus: structure, avionics, power, comms, attitude control; add propulsion, payload, entry, crew as needed)
3. **Plan your trajectory** — From the Physics Handbook
4. **Calculate your budget** — Hardware, operations, insurance (See Shop: Operations)
5. **Find your market** — Price against Shop alternatives, or argue a new product/customer with the other player (See Shop: Markets & Competition)
6. **Commit resources** — Spend the money, lock parameters and success criteria
7. **Advance time** — Pay ongoing costs through lead times, builds, and waits; hit the launch window or wait for the next
8. **Roll outcomes** — Launch, then mission (per Chance)
9. **Resolve** — Success, failure, partial; update reputation and the world
10. **Iterate** — Next mission, next year, next move

There is no winning. There is only the satisfaction of watching the numbers work—or the humility of watching them fail.

2.7 END GAME

Bust: Your company goes under when cash hits zero and you can’t argue a valuation.

But the universe continues.

- **Start over** — Found a new company. Buy the scraps of your old one at liquidation prices. Carry on with lessons learned.
- **Inherit the world** — Infrastructure you built still exists. Services you offered were real.

Companies end. The solar system doesn’t.

3. The Shop

Everything you can buy, build, or research. Prices are base costs; see Volume Discounts for production scaling.

READING THIS CATALOG

- **F** — Addressable failures per 1000 days of relevant effort (for engines: burn-days while thrusting). Count modes that force a player response, not only total loss. Lower = more reliable. See Chance; propulsion header has the burn-day / † split.
- **†** — One-shot item. Number is % failure per activation (not per time). Roll d100 at activation; fail if roll < † (see Chance: One-Shot Components). Prefer † when risk is at start/fire rather than during run time.
- → **Italics** — Odds chapter for failure modes. Items with both † and F use **two** Odds entries: † → activation table (†); F → continuous table (F). Never roll ignition/deploy modes from burn-day F.
- **D; P** — Research required. D = difficulty (\$), P = probability it's possible. See Chance: Research. *Items with D factors are not available for purchase until research breakthrough is completed (by you or another player). List price shown is what it will cost after research.*
- **Tier (T1–T5)** — Manufacturing complexity. T5 = precision (electronics, optics, engines): buy on Earth, or build in a Precision Fab; in space, first five units of a new design are prototypes (×2 F) — see Making Stuff.
- **N/yr** — Units produced per year in the world. Drives lead time and market size.
- **Flags** — Special procurement restrictions: **Licensed** (government/regulatory approval required; +6–12 mo), **Captive** (produced but not sold externally; must develop own or negotiate access), **Single-source** (one supplier; +6 mo, pricing leverage). *Bracket tag after flags = supplier identity used for procurement rolls: [Maker] for US, [Maker, XX] for non-US (2-letter country code), ; ITAR if export-controlled (US missile-tech rules apply to foreign buyers).*
- **kWh (electrical)** — At continuous rated power over one Earth day: **kWh/day** = **(kW) × 24** (equivalently **kWh/day** = **(W/1000) × 24**). **Electric thrusters** draw at that rate only while thrusting at full rated power. **Intermittent** loads: **kWh** = **(kW) × (hours at rated power)** (e.g. 25 kW for 0.5 h → 12.5 kWh). **Generators** (solar, reactor, RTG, fuel cell) list **kWh/day output** at rated conditions, not bus draw. Components without a listed electric draw have **no kWh in this table** until you assign a drive rating; then use the same formulas.

LEAD TIME (from N/yr)

N/yr	Lead Time	Interpretation
1000+	Days–weeks	Off the shelf
100–999	1–3 months	Catalog order
10–99	3–12 months	Production queue
1–9	12–24 months	Built to order
<1	24–36+ months	Mission-specific

PROCUREMENT FLAGS — DETAILED PLAY

The flag tells you the *shape* of the obstacle. Argue specifics from supplier identity, your reputation, and the political moment.

- **Licensed** — Approval is a roll, not just a delay. Filing costs \$1–5M plus the +6–12 mo lead. Roll d100 against an argued DC (standard friendly jurisdiction = 90; novel mission = 75; nuclear deep-space = 75; nuclear surface remote (Moon/Mars) = 60; nuclear Earth-orbit = 50; nuclear Earth surface = 20; export to non-treaty ally = 30; dual-use to strategic rival = 5). Modifiers: reputation ±5/step, prior incident in this category -20, national-interest framing +5–15. Above DC+20 → denied; re-apply after 12 mo + material change. Annual revocation risk: $d100 \leq 2$ (≤ 5 for

nuclear) → 6–12 mo suspension pending review.

- **Captive** — N/yr available externally = 0. Three paths:
 - *Develop your own* at D_{dev} (standard rules).
 - *Negotiate access* at 2–5× list with non-compete strings — roll $d100 \leq 70$ baseline; -20 if you compete with the maker’s main business, +10 per reputation step above Established, +20 for equity offer. Fail = no deal; partial = strings (volume cap, no-resale, ROFR on your launches).
 - *Acqui-hire / equity* — board seat, exclusivity, or strategic concession in exchange for guaranteed supply.
- **Single-source** — Spot price = list × 1.0–1.5× depending on the supplier’s order book; no volume discount from their side. Government customers preempt commercial (+6–12 mo if behind a NASA/DoD priority line). Supply-disruption roll: each year $d100 \leq 5 \rightarrow 12\text{-mo disruption} (\leq 15 \text{ for strategic-rival or sanction-vulnerable sources})$. Mitigations: long-term contract halves the roll; equity stake or strategic investment zeroes it; *qualifying a second source* costs full D_{dev} but converts you to dual-source.

QUALIFYING A SECOND SOURCE — Aerospace procurement term: paying to engineer, build, and certify an alternative supply of an equivalent part so your vehicle can fly with either one. In game: spend D_{dev} on the alternative item (or develop a clean-sheet equivalent), run a requalification campaign (folded into the dev cost), and from then on either source satisfies your design. Disruption rolls now only bite if *both* sources go down simultaneously (treat as independent). This is also how you escape a Captive supplier’s leverage long-term: qualify an alternative, then renegotiate from a position where you don’t need them.

DEVELOPMENT & IMPROVEMENT — See Volume Discounts & Manufacturing. Quick ref: $D_{\text{dev}} = \text{tier mult} \times \text{list}$ (T1–T2: 2×, T3: 3×, T4: 5×, T5: 10×); pay D_{dev} **plus** lab rent/ops; min 1 year. D; P items need research first. Improve: $D_{\text{dev}} \times (X/25)^2$.

3.1 LAUNCH VEHICLES

Pre-built rockets purchased as launch services. Price includes hardware, propellant, operations, and provider risk.

Class	Payload to LEO	Price	Example	Upper Stage	†Launch
S-class	250 kg	\$8M	Electron	Kerolox	5%
M-class	22,000 kg	\$70M	Falcon 9	Kerolox	1%
H-class	64,000 kg	\$120M	Falcon Heavy	Kerolox	2%
X-class	150,000 kg	\$200M	Starship	Methalox	5%

Launch failure: Everything on the rocket is destroyed — loss of launch cost and payload. Nothing is covered unless insured or contractually guaranteed (negotiate re-fly clauses via Contracts if desired). Add †Launch to mission risk as a one-shot (see Chance).

3.1.1 PAYLOAD TO OTHER DESTINATIONS

LEO payload is the baseline ($\Delta v_{\text{LEO}} \approx 9.2 \text{ km/s}$ from Earth’s surface). For other destinations, payload decreases because more delta-v means more fuel mass, leaving less room for payload.

Payload for extra Δv : For additional delta-v beyond LEO, payload scales as:

$$m_{\text{payload}} = m_{\text{LEO}} \times e^{-\Delta v_{\text{extra}}/v_e}$$

Where $v_e = \text{Isp} \times 9.81 \text{ m/s}$. For kerolox upper stages (Isp 310s), $v_e \approx 3.0 \text{ km/s}$.

Quick reference (conservative estimate, Isp 310s / $v_e = 3.0 \text{ km/s}$):

Extra Δv	Payload Factor	Example Destination
+1 km/s	0.72×	Higher LEO
+2 km/s	0.51×	—
+3 km/s	0.37×	Moon orbit, Earth escape
+4 km/s	0.26×	GEO, Mars departure
+5 km/s	0.19×	Vesta, Ceres
+6 km/s	0.14×	Moon surface, Jupiter

Table is conservative. Use the formula with actual upper stage I_{sp} (see table above) for better estimates. See *Physics Handbook: Δv Quick Reference* for destination values.

Example: M-class (22t to LEO) to GEO. Extra $\Delta v = 3.9$ km/s. Payload = $22,000 \times e^{(-3.9/3.0)} = 22,000 \times 0.27 \approx 6,000$ kg.

3.1.2 RIDESHARE

Pay per kg on a shared launch. Use the launcher that serves your destination; price = launcher cost $\times$ 1.3 markup $\times$ (your mass / launcher payload to destination).

$$\text{Cost} = \frac{\text{Launcher price} \times 1.3 \times m_{\text{yours}}}{\text{Launcher payload to destination}}$$

Launcher payload to destination = LEO capacity $\times$ payload factor (see table above). Pick smallest launcher whose payload to destination $\geq$ your mass.

Inclination: You don't choose it—you get the mission's orbit. **Wait for band:** Choose SSO, GEO, or MEO; you get that band's inclination (98°, 0°, 55°). **Take next LEO:** Roll d100. 1–35 $\rightarrow$ 53° (Starlink); 36–55 $\rightarrow$ 43° (Starlink); 56–75 $\rightarrow$ 70° (Starlink); 76–100 $\rightarrow$ 98° (Transporter, EO, recon, science). See *Astrodata: Earth Orbital Regimes*. Wrong inclination = plane change (*Physics Handbook*) or accept the orbit.

Small payloads (<50 kg) may face a slot floor (~\$200–300k) in practice; negotiate. Availability: LEO/SSO frequent; GEO/MEO rare. See Astrodata: Earth Orbital Regimes for traffic by band. †Launch risk applies from the vehicle you're riding on.

3.2 PROPULSION

Boiloff (cryo propellants): Kerolox 0.5%/day (LOX only). Methalox 1%/day. Hydrolox 3%/day. Active Cooling reduces to 0.5%/day (any). Storable/room temp: 0%.

N/yr is world production for that engine line (drives lead time). Captive lines still list N/yr as production volume; external buy availability is 0 — see Procurement Flags.

KEROLOX — I_{sp} 310s; 0.9 kg/L; \$3/kg; cryo (LOX); T5 $\rightarrow$ Chemical engine failure (F); † $\rightarrow$ Chemical ignition failure

- **Rutherford** — 24 kN; 35 kg; \$50k; 0.25%† per ignition + F:900; N/yr: 50. Captive [Rocket Lab]
- **Merlin 1D** — 845 kN; 470 kg; \$1M; 0.1%† per ignition + F:120; N/yr: 100. Captive [SpaceX]
- **RD-180** — 4,150 kN; 5,480 kg; \$15M; 0.5%† per ignition + F:1500; N/yr: <1. Single-source [Energomash, RU]
- **YF-100** — 1,200 kN; 1,920 kg; \$4M; F:250; N/yr: 80. Captive [CASC, CN]

METHALOX — I_{sp} 350s; 0.8 kg/L; \$1/kg; cryo (1%/day boiloff); T5 $\rightarrow$ Chemical engine failure (F); † $\rightarrow$ Chemical ignition failure

- **Raptor** — 2,300 kN; 1,600 kg; \$1M; 0.5%† per ignition + F:150; N/yr: 500. Captive [SpaceX]
- **BE-4** — 2,450 kN; 2,000 kg; \$15M; 0.5%† per ignition + F:75; N/yr: 5. Single-source [Blue Origin]

- **Prometheus** — 980 kN; 1,000 kg; \$1M; 2.5%† per ignition + F:1200; N/yr: <1. Single-source [ArianeGroup, FR]

HYDROLOX — Isp 450s; 0.3 kg/L; \$5/kg; cryo (3%/day boiloff); T5 → *Chemical engine failure* (F); † → *Chemical ignition failure*

- **RL-10** — 110 kN; 190 kg; \$15M; 0.25%† per ignition + F:60; N/yr: 10. Single-source [Aerojet]
- **RS-25** — 2,280 kN; 3,500 kg; \$70M; F:200; N/yr: 4. Single-source, Licensed [Aerojet; ITAR]
- **Vulcain 2.1** — 1,371 kN; 2,150 kg; \$8M; F:900; N/yr: 5. Single-source, Licensed [ArianeGroup, FR]
- **LE-9** — 1,471 kN; 2,400 kg; \$12M; F:700; N/yr: 6. Single-source [MHI, JP]

STORABLE — Isp 320s; 1.2 kg/L; \$30/kg; room temp; T5 → *Storable engine failure* (F); † → *Storable start failure*

- **AJ10** — 43 kN; 125 kg; \$5M; 0.2%† per ignition + F:5; N/yr: 5. Single-source, Licensed [L3Harris; ITAR]
- **SuperDraco** — 73 kN; 65 kg; \$2M; 0.5%† per ignition (abort-only; no burn F); N/yr: 10. Captive [SpaceX]
- **Draco** — 0.4 kN; 5 kg; \$30k; 0.2%† per activation + F:2; N/yr: 200. Captive [SpaceX]
- **R-4D-15** — 0.5 kN; 5.2 kg; \$150k; 0.1%† per ignition + F:1; N/yr: 20. Single-source, Licensed [L3Harris; ITAR]

SOLID — Isp 270s; non-restartable; price/mass includes propellant; T4 → *Solid motor failure*

- **STAR-48** — 77 kN; 2.6t wet; \$3M; 0.2%†; N/yr: 10. Single-source, Licensed [Northrop; ITAR]
- **GEM-63** — 1,650 kN; 49t wet; \$8M; 0.25%†; N/yr: 30. Single-source, Licensed [Northrop; ITAR]
- **GEM-63XL** — 2,060 kN; 53t wet; \$10M; 1.5%†; N/yr: 20. Single-source, Licensed [Northrop; ITAR]. *Vulcan strap-on; immature nozzle QC (2024–26 anomalies)*.
- **SRB** — 16,000 kN; 730t wet; \$50M; 0.1%†; N/yr: 2. Single-source, Licensed [Northrop; ITAR]. *SLS five-segment class (post-RSRM)*.
- **P120C** — 4,500 kN; 155t wet; \$8M; 0.3%†; N/yr: 18. Single-source [Avio, IT]

MONOPROPELLANT — Isp 220s; 1.0 kg/L; \$20/kg; room temp; T5 → *Simple thruster failure (monoprop)*

- **MR-103** — 1 N; 0.3 kg; \$50k; F:20; N/yr: 500. Licensed [L3Harris; ITAR]
- **MR-106L** — 22 N; 0.6 kg; \$80k; F:35; N/yr: 200. Licensed [L3Harris; ITAR]
- **MR-104** — 445 N; 1.9 kg; \$200k; F:40; N/yr: 50. Licensed [L3Harris; ITAR]

COLD GAS — Isp 70s; 0.8 kg/L; \$0.1/kg; room temp; T2 → *Simple thruster failure (cold gas)*

- **N₂ thruster (CubeSat)** — 0.1 N; 0.2 kg; \$10k; F:30; N/yr: 2000
- **N₂ thruster (flagship)** — 1 N; 1 kg; \$100k; F:8; N/yr: 100

ELECTRIC (ION) — Isp 4000s; 1.6 kg/L; \$2k/kg; room temp; T5 → *SEP thruster failure (ion)* (F)

- **NEXT-C** — 0.24 N; 50 kg; 7 kW; \$12M; F:0.5; N/yr: <1. Single-source, Licensed [Aerojet; ITAR]
- **T6** — 145 mN; 8.3 kg; 4.6 kW; \$8M; 0.5%† per restart → *SEP restart failure* + F:0.35; N/yr: <5 — Isp 4300s. Single-source, Licensed [QinetiQ, UK]

ELECTRIC (HALL) — Isp 1600–2500s; 1.6 kg/L; \$2k/kg; room temp; T5 → *SEP thruster failure (Hall)* (F)

- **SPT-100** — 80 mN; 4 kg; 1.4 kW; \$1M; F:0.25; N/yr: 50 — Isp 1600s. Single-source [Fakel, RU]
- **PPS-1350** — 90 mN; 5 kg; 1.5 kW; \$2M; F:0.3; N/yr: 30 — Isp 1660s. Single-source [Safran, FR]
- **BHT-200** — 13 mN; 1 kg; 0.2 kW; \$0.3M; F:0.6; N/yr: 100 — Isp 1390s. Licensed [Busek; ITAR]
- **Starlink Hall** — 170 mN; 2 kg; 4 kW; \$0.2M; F:0.35; N/yr: 5000 — Isp 2500s, argon. Captive [SpaceX]

RESEARCH PROPELLANTS — Not yet available; requires R&D or extraction.

Format: Isp (with appropriate engine); density; cost; storage — D; P — notes

- **Metallic Hydrogen** — Isp 1700s; 70 kg/L; \$100/kg; cryo, may destabilize — D:\$5B; P:0.3 — stable form uncertain
- **Antimatter** — Isp 10,000–100,000s; magnetic trap; \$1T/g — D:\$50B; P:0.3 — production at scale
- **Exotic Matter** — negative energy; unknown; unknown — D:\$100B; P:? — unlocked by Alcubierre (P revealed then)

RESEARCH ENGINES — Not yet flight-ready; requires R&D investment.

Format: thrust; mass; cost; F — D; P — notes. Propellant determines Isp.

NUCLEAR THERMAL — Isp 900s; uses LH₂ (\$5/kg); cryo (3%/day boiloff); T5 → *Nuclear thermal failure* (F); † → *NTR start failure*

- **NERVA-type** — 300 kN; 3,000 kg; \$50M; 3%† per cold start + F:20 — D:\$1B; P:1 — tested 1960s, re-engineering
- **Nuclear Lightbulb** — 150 kN; 4,000 kg; \$80M; F:12 — D:\$3B; P:0.8 — contained plasma, Isp 1500s

ELECTRIC (HIGH POWER) — Requires space-rated reactor or large solar arrays; T5.

- **VASIMR** — 6 N; 300 kg; 200 kW; \$10M; F:8 — D:\$500M; P:0.95 — Isp 3,000–30,000s variable
- **MPD Thruster** — 20 N; 100 kg; 500 kW; \$15M; F:25 — D:\$800M; P:0.85 — Isp 3,000s; high thrust electric

NUCLEAR ELECTRIC — Isp 4,000s+; onboard fission reactor powers electric propulsion; T5.

- **Kilopower Cluster** — 1 N; 1,500 kg; \$200M; F:6 — D:\$3B; P:0.9 — 10 kW reactor + ion
- **Megawatt-class NEP** — 50 N; 20,000 kg; \$2B; F:15 — D:\$8B; P:0.7 — **1 MW** electrical reference; enables outer planet missions

FUSION — Isp 10,000–100,000s; uses D-T (\$10k/kg) or D-He3 (\$1M/kg); cryo; T5.

- **Magnetic Confinement** — 100 kN; 100,000 kg; \$5B; F:40 — D:\$20B; P:0.5 — tokamak-derived
- **Inertial Confinement** — 50 kN; 80,000 kg; \$4B; F:50 — D:\$25B; P:0.4 — laser-driven pellets
- **Z-Pinch** — 200 kN; 60,000 kg; \$3B; F:45 — D:\$15B; P:0.45 — pulsed plasma compression

ANTIMATTER — Isp depends on design; uses antimatter propellant (requires AM production breakthrough); T5.

- **AM Catalyzed Fission** — 500 kN; 5,000 kg; \$500M; F:35 — D:\$5B; P:0.6* — *given AM supply
- **AM Thermal** — 1,000 kN; 8,000 kg; \$1B; F:40 — D:\$10B; P:0.5* — Isp 2,000s; AM heats H₂
- **AM Annihilation** — 10 kN; 2,000 kg; \$2B; F:50 — D:\$15B; P:0.4* — Isp 100,000s; direct thrust

NUCLEAR PULSE — Isp 3,000–10,000s; uses nuclear charges; politically restricted; T4 (charges T5).

- **Orion (Small)** — 10 MN; 100,000 kg; \$1B; F:5 — D:\$5B; P:0.95 — 0.1 kt charges; treaty issues
- **Orion (Large)** — 100 MN; 1,000,000 kg; \$10B; F:3 — D:\$8B; P:0.9 — interplanetary battleship

NON-REACTION — No propellant expended; Isp effectively infinite; T4 (delicate films, deployment mechanisms).

- **Solar Sail** — 1 mN/100m²; 0.01 kg/m²; \$1k/m²; 8%† per deploy → *Sail deploy failure*; F:2 → *Solar sail failure* — D:\$50M; P:1 — demonstrated (IKAROS)
- **Beamed Sail (Laser)** — 100 mN/m²; 0.01 kg/m²; \$2k/m²; F:12 — D:\$500M; P:0.9 — needs \$10B+ ground laser
- **Beamed Sail (Microwave)** — 10 mN/m²; 0.05 kg/m²; \$500/m²; F:10 — D:\$300M; P:0.9 — needs orbital transmitter

SPECULATIVE — Theoretical; may reveal new physics or requirements.

- **Bussard Ramjet** — collects H₂; thrust?; 50,000 kg; F:? — D:\$30B; P:0.1 — drag may exceed thrust
- **Alcubierre/Warp** — FTL; specs unknown — D:\$10B; P:0.05 — breakthrough reveals exotic matter requirements
- **EmDrive/Mach Effect** — reactionless?; specs unknown — D:\$500M; P:0.01 — probably doesn't work

3.3 EQUIPMENT

Format: price; mass; function; F → Odds entry — recipe, tier; N/yr; flags. Notation defined in READING THIS CATALOG.

Recipes: Percentages are **by weight** of final product. Items marked (T5) require Precision Fab or Earth purchase. Generic terms: **metal** = any structural metal; **chemicals** = appropriate chemical from Materials section.

Equipment Market Reference (*E* values for demand formula; see *Markets & Competition*)

Category	E	Notes
Required for launch	0.2	Must have; no substitution
Attitude control	0.5	Limited substitution between types
Cargo & docking	0.5	Mission-dependent
Communications	0.5	Choose band, but need comms
Crew systems	0.1	Life or death; nearly inelastic
Entry & landing	0.3	Safety-critical
Guidance & navigation	0.3	Safety-critical
Power	0.5	Size to mission; some source flexibility
Resource extraction	0.7	Infrastructure investment; price-sensitive
Robotics & assembly	0.8	Discretionary capability
Observation payloads	1.0	Mission-driven; science elastic, operational data less so
Surface operations	1.0	Discretionary
Tethers & cables	0.7	Niche applications
Thermal	0.4	Required but sized to mission

REQUIRED FOR LAUNCH

Typical bus: structure + avionics + power + comms + attitude control; add propulsion/tanks, payload, entry/landing, or crew as needed. Full builds → Exercises.

- **Avionics (Cubesat)** — \$25k; 0.5 kg; 2 W; F:0.5 → *Avionics failure* — T5; N/yr: 2000. *Simple LEO / rideshare bus, minimal autonomy. Flight computer + command/data handling; commercial parts OK (higher F).*
- **Avionics (Smallsat)** — \$150k; 5 kg; 25 W; F:0.2 → *Avionics failure* — T5; N/yr: 500. *Dedicated small bus, modest payload ops.*
- **Avionics (Spacecraft)** — \$500k; 50 kg; 200 W; F:0.1 → *Avionics failure* — T5; N/yr: 500. *Flagship, deep-space, or high-reliability command/data handling.*
- **Avionics (Station)** — \$5M; 200 kg; 1 kW; F:0.05 → *Avionics failure* — T5; N/yr: 20. *Habitat, base, or multi-crew facility control.* Licensed [various; ITAR]
- **Ground contact** — LEO short campaign \$25k; LEO network \$100k/yr lights-out / \$500k/yr duty per program; deep-space tracking \$3M/yr cruise / \$8M/yr hot. People → Personnel. See Operations.
- **Spacecraft Structure** — \$100k/t; 10% of dry mass (excluding propellant); F:0.05 → *Structure failure* — 100% metal, T2; N/yr: 1000+. *Framework, mounting points, rails.*
- **Tanks** — \$100k/t; 12% of propellant mass ($s=0.12$); F:0.05 → *Structure failure* — 100% metal, T2; N/yr: 1000+

ATTITUDE CONTROL

- **Electromagnet (Actuator)** — \$50k; 0.5 kg; 1 N; 2 W continuous; F:0.4 → *Actuator failure* — 60% Cu + 30% Fe + 10% insulation(T5), T4; N/yr: 1000+. *Valves, small mechanisms.*
- **CMG (Control Moment Gyro)** — \$10M; 200 kg; 2000 N·m·s, 100 N·m torque; 0.5 kW; F:0.4 → *CMG failure* — T5; N/yr: 20. *Station-class.* Licensed [Honeywell; ITAR]
- **Reaction Wheel (small)** — \$0.5M; 5 kg; 1 N·m·s capacity; 50 W; F:0.3 → *Reaction wheel failure* — T5; N/yr: 5000. *Cubesat/smallsat.*
- **Reaction Wheel (standard)** — \$2M; 30 kg; 50 N·m·s capacity; 100 W; F:0.2 → *Reaction wheel failure* — T5; N/yr: 500. *Typical spacecraft.*
- **RCS Thruster Block** — \$1M; 20 kg; 4× monoprop thrusters; F:0.5 → *RCS failure* — T5; N/yr: 500. *Includes valves, lines. Propellant separate.*

CARGO & DOCKING

- **Cargo Storage** — \$0.1M/t; 5× capacity; F:0.02 → *Structure failure* — 100% metal, T2; N/yr: 500
- **Docking Hardware** — \$1M/t; 0.2t; refuel (10% loss); 1%† → *Seal failure* — 80% metal + 15% seals + 5% mechanisms(T5), T3; N/yr: 50

- **Electromagnet (Docking)** — \$10M; 250 kg; 500 N; 25 kW during approach (25 kWh per hour at rated approach load); F:0.4 → *Docking hardware failure* — T4; N/yr: 5. *1 ton package capture. Use latching for hold.*
- **Electromagnet (Latching)** — \$20M; 250 kg; 500 N; 0 W hold; 1%† per latch cycle → *Mechanical failure* — T4; N/yr: 5. *Permanent magnet + pulse coil. For long-term docking.*
- **Separation System** — \$100k; 5 kg; pyrotechnic or non-explosive release; 0.2%† → *Separation failure* — 70% metal + 20% pyrotechnic/actuator(T5) + 10% wiring(T5), T3; N/yr: 1000+
- **Structural Docking** — \$50k/t; 0.1t; F:0.01 → *Structure failure* — 100% metal, T2; N/yr: 100
- **Propellant Tank (orbital)** — \$2M/t capacity; 0.1t dry per 1t capacity; F:0.05 → *Propellant tank failure* — D:\$200M; P:1.0 — T2; N/yr: <1. *Same boiloff as propellant type. Includes docking interfaces, transfer systems, long-duration certification.*
- **Transfer Pump** — \$1M; 0.2t; 1t/hr transfer; 5% loss; F:0.3 → *Transfer pump failure* — T3; N/yr: 20. *Replaces docking hardware loss if present.*

COMMUNICATIONS

- **UHF/S-band** — \$100k; 3 kg; 10 W; 1 kbps at 1 AU; F:0.15 → *Communications failure* — T5; N/yr: 2000. *Low-rate.*
- **S-band** — \$300k; 8 kg; 20 W; 10 kbps at 1 AU; F:0.15 → *Communications failure* — T5; N/yr: 1000. *Standard.*
- **X-band** — \$1M; 15 kg; 40 W; 1 Mbps at 1 AU; F:0.2 → *Communications failure* — T5; N/yr: 200. *High-rate.*
- **Ka-band** — \$2M; 25 kg; 80 W; 10 Mbps at 1 AU; F:0.3 → *Communications failure* — T5; N/yr: 100. *Highest rate.*
- **High-Gain Antenna** — \$1M/m²; 5 kg/m²; 1 W pointing; ×10 data rate per m²; F:0.1 → *Antenna failure* — 80% composite + 20% motors(T5), T3; N/yr: 50
- **DSN Time** — \$1k/hr — ground antenna time for direct-to-Earth. Required beyond ~20 light-minutes. Limited availability (negotiate schedule). Single-source [NASA/JPL]
- **Relay Satellite** — \$10M/t; 0.3t; F:0.1 → *Communications failure* — 30% structure + 40% antenna + 30% electronics(T5), T5; N/yr: 5

Power and rate are for direct-to-Earth at 1 AU (e.g. 10 W → 0.24 kWh/day → 1 kbps). Received power (link margin) scales as $1/r^2$ where r = distance to receiver (AU). Relay satellites are typically much closer than 1 AU, so the same hardware can achieve higher data rates over the relay link; the gain in rate is logarithmic in the power gain, not proportional to $1/r^2$.

CREW SYSTEMS

- **Closed Biosphere** — \$100M; 50t; 100% recycling; indefinite crew duration; D:\$2B; P:0.6 — hard problem
- **Cryosleep Pod** — \$10M; 0.5t/crew; no consumables during transit; D:\$5B; P:0.3 — induced torpor
- **Life Support** — \$2M/t; 1t/crew + 5kg/crew/day; 2 kW/crew (48 kWh/day per crew); F:0.5 → *Life support failure* — 40% metal + 30% chemicals + 20% electronics(T5) + 10% bio, T4; N/yr: 20
- **Magnetic Rad Shield** — \$50M; 5t; reduces radiation F by ×0.5 (replaces 10t passive); D:\$500M; P:0.7 — superconducting magnets. *See Astrodata: Environmental Hazards.*
- **Radiation Hardening** — 2× component cost; no mass penalty; reduces electronics radiation F by ×0.3; N/yr: 200 — Applied at purchase to any T5 electronics. Stackable with shielding (×0.5 × ×0.3 = ×0.15). *See Astrodata: Environmental Hazards.*
- **Radiation Shielding** — \$1M/t; 0.5t/crew; reduces radiation F by ×0.5; F:0.01 → *Structure failure* — 100% mass (rock/water/metal), T1; N/yr: 50. *See Astrodata: Environmental Hazards for radiation zones and consequences.*
- **Spin Gravity Module** — \$20M; 10t; 2–4 crew; 0.3–1g selectable; 15 kW (360 kWh/day when spun up); F:0.8 → *Spin gravity failure*; D:\$200M; P:0.95 — 40% metal + 25% bearings(T5) + 15% seals + 10% motors(T5) + 10% controls(T5), T4. *5m radius, requires 30t counterweight. Prevents bone/muscle loss (>6 mo missions). See Physics Handbook: Artificial Gravity.*
- **Whipple Shield** — \$200k/t; 20 kg/m²; reduces debris/micrometeorite F by ×0.3; F:0.05 → *Whipple shield damage* — 60% Al + 30% Kevlar + 10% structure, T3; N/yr: 200. *Size by exposed area. See Astrodata: Environmental Hazards for debris F by location and consequences.*

ENTRY & LANDING

- **Heat Shield** — \$2M/t; 15% of entry mass; 0.5%† → *Heat shield failure* — 100% ablative (carbon/ceramic), T4; N/yr: 50
- **Landing Gear** — \$1M/t; 5% of landed mass; 2%† → *Landing gear failure* — 90% metal + 10% actuators(T5), T3; N/yr:

100

- **Parachutes** — \$0.5M/t; 3% of landed mass; 1%† → *Parachute failure* — 90% fabric + 10% cable, T4; N/yr: 100
- **Sample Return Capsule** — \$5M/t; 0.3t + 20% of sample mass; 1%† → *One-shot failure* — 70% heat shield + 30% container, T4; N/yr: 2

GUIDANCE & NAVIGATION

- **IMU** — \$1M; 2 kg; 5 W; angular rate + linear acceleration (high-rate, short-term); F:0.3 → *IMU failure* — T5; N/yr: 500. *Use for slews, maneuvers, and between fixes.*
- **Radar Altimeter** — \$3M; 8 kg; 10 W; landing/proximity ops; F:0.2 → *Guidance/navigation failure* — T5; N/yr: 50
- **Sensors** — \$5M/t; 0.2t base; 150 W at 0.2t; $\times(\text{mass}/0.2\text{t})$ for other masses; detection range = $1000\text{km} \times \sqrt{\text{mass}/0.2\text{t}}$; search volume = $\pi \times \text{range}^2 \times \text{path length}$; F:0.2 → *Guidance/navigation failure* — 20% structure + 40% optics(T5) + 40% electronics(T5), T5; N/yr: 100
- **Sun Sensor** — \$50k; 0.1 kg; 1 W; 0.5° attitude; F:0.1 → *Sensor drift* — T4; N/yr: 2000. *Coarse backup; solar-powered craft only.*
- **Star Tracker** — \$2M; 3 kg; 10 W; 0.001° attitude knowledge; F:0.2 → *Star tracker failure* — T5; N/yr: 500. *Minimum 2 for redundancy.*
- **Terrain-Relative Nav** — \$5M; 5 kg; 35 W; autonomous hazard avoidance; F:0.4 → *Software fault* — T5; N/yr: 10. *Required for precision landing.*

POWER

- **Battery Pack** — \$0.5M/t; 0.2t/kWh; F:0.06 → *Battery failure* — 80% lithium + 20% electronics(T5), T4; N/yr: 1000+. *Standard space-rated lithium-ion.*
- **Battery - High-Energy** — \$2M/t; 0.1t/kWh; F:0.04 → *Battery failure* — 75% advanced lithium + 25% electronics(T5), T5; N/yr: 200. *Higher energy density; lighter weight.*
- **Battery - Thermal-Hardened** — \$1M/t; 0.2t/kWh; F:0.03 → *Battery failure* — 70% lithium + 20% thermal management + 10% electronics(T5), T4; N/yr: 50. *Operates -180°C to +120°C; lunar day/night capable.*
- **Fission Reactor** — \$200M; 2t; 100kW for 10 yr (2,400 kWh/day output at rated 100 kW); F:0.3 → *Reactor failure* — nuclear + shielding + turbomachinery(T5), T5; N/yr: <1. *Licensed [DOE-licensed]*
- **Fuel Cell** — \$3M/t; 0.1t/kW; 24 kWh/day output per kW nameplate at continuous full output (consumes H₂/O₂ at 0.5 kg/kWh); F:0.4 → *Fuel cell failure* — 70% metal + 30% membranes(T5), T4; N/yr: 20
- **RTG (Radioisotope thermoelectric generator)** — \$50M; 0.05t; 0.5kW for decades (12 kWh/day output at rated 0.5 kW); required past ~5 AU; F:0.01 → *Electronic failure* — nuclear + thermocouples(T5) + shielding, T5; N/yr: <1. *Licensed, Single-source [DOE]*
- **Solar Panel** — \$300k/m²; 3.5 kg/m²; 330 W/m² at 1 AU (7.92 kWh/m²·day output at continuous full sun, 330 W), scales as $1/r^2$; 1%† deployment → *Deployment jam (panels)*; F:0.15 → *Solar panel failure* — 70% Si + 20% Al + 10% wiring(T5), T3; N/yr: 5000+
- **Solar Panel Tracking (Small)** — \$20k; 2 kg; max I = 10 kg·m²; 5 W continuous drive (0.12 kWh/day at full drive duty); F:0.2 → *Actuator failure* — T3; N/yr: 5000. *Rotates panels to face sun.*
- **Solar Panel Tracking (Medium)** — \$50k; 5 kg; max I = 50 kg·m²; 10 W continuous drive; F:0.2 → *Actuator failure* — T3; N/yr: 500. *Rotates panels to face sun.*
- **Solar Panel Tracking (Large)** — \$100k; 10 kg; max I = 200 kg·m²; 20 W continuous drive; F:0.2 → *Actuator failure* — T3; N/yr: 100. *Rotates panels to face sun.*

Quiescent/hold power may be lower; values above are continuous slewing at rated drive power.

REPAIR & SPARES

- **Cannibalize** — Strip working system to repair another. Donor destroyed; no spares needed; 90% success.
- **Repair** — Requires spares + crew (or repair bot) + 1 day per \$1M component value. Success: 80% (generalist), 95% (specialist), varies (bot per specs). Failure consumes spares; retry allowed.
- **Spare Parts** — 10% of component mass/cost per repair attempt; consumed on use

Uncrewed repair requires robotic arm or repair bot. Some failures are unrepairable (look for “destroyed” or “vehicle loss” in Odds entries).

RESOURCE EXTRACTION & PROCESSING

- **Electrolysis Unit** — \$5M; 1t; 10 kg H₂O/day → 8.9 kg O₂ + 1.1 kg H₂; 50 kW (1,200 kWh/day); F:1.2 → *Electrolysis failure* — 50% metal + 30% membranes(T5) + 20% controls(T5), T4; N/yr: 50
- **Lunar He-3 Processor** — \$50M; 20t; 1 kg He-3/yr per unit (processes 50kt regolith/yr); D:\$500M; P:0.95 — lunar surface only
- **Mining Rig** — \$20M; 2t; 1/hardness t/day (ice=1, rock=3, ore=5, metal=10); 50 kW (1,200 kWh/day); F:1 → *Mining failure* — 60% metal + 25% motors(T5) + 15% electronics(T5), T3; N/yr: 100
- **Refinery** — \$100M; 5t; 1t ore/day → 0.1t output; 100 kW (2,400 kWh/day); F:0.8 → *Refinery failure* — 50% metal + 30% reactors + 20% controls(T5), T4; N/yr: 20
- **Sabatier Reactor** — \$9M; 1.5t; 20 kg/day → 6 kg CH₄ + 14 kg H₂O (needs 3 kg H₂ + 17 kg CO₂); 20 kW; F:1.0 → *Sabatier failure* — 55% metal + 25% reactors + 20% controls(T5), T4; N/yr: 5

ROBOTICS & ASSEMBLY

- **Assembly Jig** — \$10M; 1t; holds components during construction; F:0.1 → *Mechanical failure* — 90% metal + 10% clamps(T3), T3; N/yr: 10
- **Grapple Fixture** — \$100k; 10 kg; passive capture point; F:0.01 → *Structure failure* — T3; N/yr: 50. *One per grappled object.*
- **Inspection Drone** — \$1M; 5 kg; free-flying camera; 2 hr battery; F:1 → *Drone failure* — T5; N/yr: 20
- **Robotic Arm (Light)** — \$5M; 50 kg; 2m reach; 25 kg capacity; 0.5 kW; F:0.2 → *Robotic arm failure* — T5; N/yr: 30
- **Robotic Arm (Heavy)** — \$20M; 500 kg; 10m reach; 1t capacity; 3 kW; F:0.2 → *Robotic arm failure* — T5; N/yr: 5. Single-source, Licensed [MDA, CA; ITAR]

OBSERVATION PAYLOADS

- **Camera (Engineering)** — \$300k; 2 kg; 8 W mission-average ($\approx$ 2 W standby, $\sim$ 12 W imaging/downlink bursts); navigation/documentation; 1m resolution at 1km; F:0.15 → *Camera failure* — 40% optics(T5) + 40% electronics(T5) + 20% structure, T5; N/yr: 500. *Duty-cycle load; use intermittent kWh rule for tight power budgets.*
- **Camera (Science)** — \$5M; 10 kg; 22 W (higher readout + thermal; brief peaks $\sim$ 35 W with filter wheel / full-frame); high-resolution imaging; 0.1m resolution at 1km; F:0.2 → *Camera failure* — 50% optics(T5) + 40% electronics(T5) + 10% structure, T5; N/yr: 30
- **Coronagraph** — \$15M; 20 kg; 15 W; occults solar disk to image corona; CME detection, speed, and direction; F:0.35 → *Coronagraph failure* — 50% optics(T5) + 35% electronics(T5) + 15% structure, T5; N/yr: 3. *Alignment-critical. L1 or L5 vantage for Earth-directed CME warning. Single-source [NRL]*
- **GNSS Receiver (Space)** — \$2M; 4 kg; 15 W; receives GNSS signals through atmosphere; ionospheric electron-density profiles via radio occultation; F:0.15 → *Receiver failure* — 70% electronics(T5) + 20% antenna + 10% structure, T5; N/yr: 100. *LEO only. Constellation improves temporal/spatial coverage.*
- **Ground Sensor Node** — \$100k; 0.2t; ground magnetometer + GNSS reference receiver; $\sim$ 100 km effective radius; F:0.5 → *Electronic failure* — 40% electronics(T5) + 30% structure + 20% wiring(T5) + 10% antenna, T3; N/yr: 1000+. *Ground asset (not launched). Requires site lease. Deploy as network for regional data products.*
- **Magnetometer (Science)** — \$1M; 3 kg; 3 W; ambient magnetic field to 0.1 nT; F:0.15 → *Instrument failure* — 60% electronics(T5) + 20% structure + 20% wiring(T5), T5; N/yr: 100. *Boom-mounted (+2 kg boom, 0.5% $\neq$ deployment). Planetary science, magnetospheric surveys, space weather.*
- **Particle Detector** — \$5M; 10 kg; 8 W; energetic electron/proton/ion flux and spectra; radiation environment characterization; F:0.3 → *Instrument failure* — 60% electronics(T5) + 25% structure + 15% shielding, T5; N/yr: 30. *GEO/HEO for radiation belts; any orbit for crew/hardware dosimetry.*
- **Plasma Analyzer** — \$8M; 8 kg; 10 W; solar wind speed, density, and temperature (Faraday cup / electrostatic analyzer); F:0.25 → *Instrument failure* — 65% electronics(T5) + 25% structure + 10% wiring(T5), T5; N/yr: 10. *L1 for upstream solar wind warning; interplanetary probes for heliospheric science.*
- **Research Lab** — \$20M/t; 5t; $\times 2$ effective investment for one program; 5 kW; F:0.3 → *Lab failure* — 30% metal + 40% equipment(T5) + 30% computers(T5), T5; N/yr: 2
- **Spectrometer** — \$2M; 5 kg; composition analysis; visible/near-IR; 10m range; F:0.25 → *Spectrometer failure* — 45% optics(T5) + 45% electronics(T5) + 10% structure, T5; N/yr: 50

SURFACE OPERATIONS

- **Airlock** — \$5M; 0.5t; crew EVA access; 0.5%† → *Airlock failure* — 70% metal + 20% seals + 10% mechanisms(T5), T3; N/yr: 5
- **EVA Suit** — \$2M; 120 kg; 8 hr duration; 1%† → *EVA suit failure* — 40% fabric + 30% life support(T4) + 30% joints(T5), T4; N/yr: 10. Single-source, Licensed [Collins; ITAR]
- **Lunar Humanoid Robot** — \$5M; 80 kg; general-purpose labor; 2 kW (16 kWh per 8 h shift at full load; 48 kWh/day if run continuously at 2 kW); 8 hr battery; F:0.8 → *Robotic failure*; D:\$200M; P:0.95 — vacuum seals, dust mitigation, thermal, reduced-g locomotion
- **Regolith Bagger** — \$2M; 0.5t; 10t/day shielding material; 10 kW; F:0.6 → *Mechanical failure* — 80% metal + 20% motors(T5), T2; N/yr: 5
- **Rover (Unpressurized)** — \$10M; 0.3t; 0.1 km/day; 20 km range; 50 kg payload; 0.5 kW; F:0.8 → *Rover failure* — 60% metal + 30% motors(T5) + 10% electronics(T5), T3; N/yr: 5
- **Rover (Pressurized)** — \$50M; 2t; 0.2 km/day; 100 km range; 2 crew; 5 kW; F:1.2 → *Rover failure* — includes life support + airlock, T4; N/yr: <1
- **Surface Habitat** — \$20M/t; 5t base; 4 crew; 10 kW; F:0.5 → *Surface habitat failure* — 50% metal + 30% life support(T4) + 20% structure, T4; N/yr: <1. *Add 2t/crew beyond 4.*

TETHERS & CABLES

- **Electrodynamic Tether** — \$500/m; 0.05 kg/m (5mm Al); F:0.5/km → *Tether failure* — T5; N/yr: <1. *Working load* ≈ 1 kN. *Generates ~1 mN/km thrust in LEO.*
- **Grapple/Catch Mechanism** — \$5M; 100 kg; 10t capacity; 2%† → *Grapple failure* — T5; N/yr: 5. *For skyhook operations.*
- **Tether Deployer** — \$2M; 200 kg; 1%† → *Tether deployer failure* — T5; N/yr: 5. *Spool with brake/motor. Controlled release up to 50 km. Reel in/out, tension control.*
- **Tether Hook** — \$200k; 5 kg; 1t capacity; 1%† → *Mechanical failure* — T3; N/yr: 20. *Carabiner-style connector. Attaches to grapple fixtures.*
- **Kevlar Tether** — \$50/m/mm²; 1.44 kg/m/mm²; F:0.2/km → *Tether failure* — T4; N/yr: 10 (km). *Working load* = 3.6 GPa × area / 4. *Standard space tether.*
- **Zylon Tether** — \$300/m/mm²; 1.56 kg/m/mm²; F:0.15/km → *Tether failure* — T5; N/yr: <1 (km). *Working load* = 5.8 GPa × area / 4. *Highest strength; shield from UV.*

THERMAL

- **Active Cooling** — \$1M/t; 5% of tank mass; 0.5%/day boiloff; F:0.5 → *Cooling loop failure* — 85% Al + 10% pumps(T5) + 5% ammonia, T3; N/yr: 50
- **Cryocooler** — \$2M; 50 kg; 10W cooling at 80K; 2 kW power; F:0.4 → *Cryocooler failure* — 50% metal + 30% compressor(T5) + 20% controls(T5), T5; N/yr: 100. Single-source, Licensed [Northrop; ITAR]
- **MLI Blankets** — \$50k/t; 1 kg/m²; passive thermal control; F:0.02 → *Thermal failure* — 100% mylar/dacron, T4; N/yr: 1000+
- **Radiator Panels** — \$0.5M/t; 10 kg/kW rejected; F:0.1 → *Radiator failure* — 90% Al + 10% pumps, T3; N/yr: 500. *Required: 1 kW rejection per 1 kW generated.*

MEGA-INFRASTRUCTURE — Planet-scale projects; civilization-changing megaprojects; requires R&D investment.

- **EM Launcher** — \$5B; 1kt/day to orbit; high-g only (raw materials); D:\$10B; P:0.85 — lunar/asteroid surface
- **Launch Loop (Lofstrom)** — \$30B; ~\$50/kg to orbit; 5kt/day capacity; D:\$20B; P:0.7 — active structure, 80km altitude, magnetic rotors
- **Space Elevator (Earth)** — \$100B; ~\$10/kg to orbit; 10kt/day capacity; D:\$50B; P:0.5 — carbon nanotube cables

3.4 CREW

Flight staff, crew counts, specialists, and rotation → **Personnel**. Remote ops people → Personnel (ops attention); antennas/network → **Ground contact** below.

3.5 OPERATIONS

Remote ops is **people + contact**. There is no flat \$/mission mission-control fee.

PEOPLE — Assign Ground to **ops** (Personnel). Each provides **1.0** ops attention. Founder counts. 1-month onboard. Ops Ground may not run a spin-up the same month, except **lights-out** standing load may share with *idle* lab/factory work in months with no peak. Pay is ordinary Ground pay (\$250k/yr) — do not add a separate MC headcount line.

Group active craft into **ops programs** (same architecture + regime). Each program has standing load ℓ . Require $\sum \ell \leq$ ops attention.

Class	ℓ	Use when
Lights-out LEO	0.25	LEO store-and-forward / weekly-ish contacts; no customer refund SLA
Duty LEO	1.0	Daily contacts, commercial SLA, or refund SLA
Cruise	0.5	Beyond LEO / long coast (one craft per program)
Hot — encounter / entry / dock	3	While that campaign runs, then drop back
Hot — crewed remote support	4	While crewed remote support is active
Hot — open crisis	6	While a serious Odds cascade is open

Safed or finished: $\ell = 0$. **Copies** on the same lights-out program: +0 while $N \leq 12$; at $N = 13$ open a second lights-out program (+0.25) or bump the program to Duty. Same duty program: +0 while $N \leq 6$; at $N = 7$ open a second duty program (+1.0). Cruise: no free copies — each craft is its own program at 0.5.

Peaks — Critical windows need p people on ops that week (same roster; they focus, they don't clone). Overlapping peaks add. If p exceeds ops roster: delay, hire, borrow, or accept degraded response.

Peak	p	Duration
Rideshare + early ops, lights-out, no refund SLA	1	2 weeks
Rideshare + early ops, duty / SLA / refund	2	2 weeks
You operate the launch vehicle	6	2 weeks
Commanded burn / dock / entry, interactive / contingency live	3	3 days
Commanded burn, pre-loaded burn-asap LEO, no contingency branch	2	3 days
Odds cascade (serious)	+2	while open

Borrow — Named peak window only. Cost $2\times$ pro-rated Ground pay for those weeks. Onboard waived if contractor; full 1 month if roster hire. Counts toward p only during the window.

Customer-furnished ops — If the customer already runs an adequate program + contact for the craft class: you pay **\$0** contact and **\$0** standing ℓ . You still need peak p on *your* roster the week you command a burn or own a success gate.

GROUND CONTACT — Antenna / network time (Shop). Stand up **1 month** before first contact; cannot buy day-of.

Line	Cost	Term
LEO short campaign	\$25k flat	One lights-out program; ≤ 90 days first contact $\rightarrow$ safe
LEO network, lights-out	\$100k/yr per program	Pro-rate by month; min 3 months
LEO network, duty	\$500k/yr per program	Pro-rate by month; min 3 months
Deep-space tracking, cruise	\$3M/yr	Min 1 year
Deep-space tracking, hot	\$8M/yr	Min 1 year ; use for encounter / continuous high-rate

CONSUMABLES — Life support: $5 \text{ kg/crew/day} \times \$100/\text{kg} = \$500/\text{crew/day}$. Propellant boiloff per tank specs.

INSURANCE — Optional. Premium = $(1.5 \times \text{total risk} + 0.01) \times \text{insured value}$. The $1.5\times$ is the insurer's margin and loss reserve; the 0.01 is the underwriting fee, charged per policy (mission insurance: per year covered). Total risk = $\Sigma(F \times \text{time}) + \Sigma(\text{one-shot } \uparrow)$ for covered hardware, calculated per Chance. Insured value = list price of covered hardware. Payout: list price on total loss; partial loss proportional to systems destroyed. Not available when total risk > 0.3 .

Launch insurance: Covers launch cost + payload value during launch. Risk = $\uparrow$ Launch from vehicle table. Insured value = launch cost + payload list price.

Mission insurance: Covers post-separation operations. Risk = mission total from Chance. Purchase per mission phase or per year for long-duration operations.

Hardware degradation and repair are handled by F values and Repair & Spares — no separate maintenance cost. Spares for specific failures are listed in Odds entries.

Example: Falcon 9 launch (\$70M) of \$20M spacecraft on a 3-year cruise (mission risk ≈ 0.08). Launch insurance: $(1.5 \times 0.01 + 0.01) \times (\$70M + \$20M) = 0.025 \times \$90M = \$2.25M$. Mission insurance: $(1.5 \times 0.08 + 3 \times 0.01) \times \$20M = 0.15 \times \$20M = \$3.0M$. Ops: 1 Ground on cruise ($\$250k \times 3 = \$0.75M$) + deep-space cruise contact ($\$3M \times 3 = \$9M$) = $\$9.75M$. Total operations $\approx \$9.75M + \$5.25M$ insurance = $\$15M$. Uninsured launch failure: $\$90M$ lost.

3.6 MATERIALS

Prices shown are Earth prices (E:). In space, price = Earth price + launch cost to destination. Annual Earth market size in parentheses. Selling $>10\%/yr$ of Earth market drops price 20%; stacks; recovers 10%/yr.

Material Elasticity (E values for demand formula; see Markets & Competition)

Category	E	Notes
Bulk	0.2	Always needed; substitution rare
Structural metals	0.5	Some material substitution possible
Strategic metals	0.8	High-value; buyers shop aggressively
Volatiles	0.5	Process inputs; moderate flexibility
Organics & chemicals	0.5	Industrial feedstock
Tether & high-strength	0.7	Material choice varies by application

BULK (T1)

- **Silicates/Rock** — E:\$0.01/kg (50Gt/yr, \$500B) — shielding, silicon feedstock
- **Clays** — E:\$0.05/kg (300Mt/yr, \$15B) — ceramics, water extraction
- **Water/Ice** — E:\$0.001/kg (4Gt/yr, \$4B industrial) — propellant, life support

STRUCTURAL METALS (T2 via refinery)

- **Iron/Nickel** – E:\$0.1/kg (2Gt/yr, \$200B) – construction, steel
- **Aluminum** – E:\$2/kg (70Mt/yr, \$150B) – lightweight structures
- **Magnesium** – E:\$3/kg (1.5Mt/yr, \$5B) – ultralight alloys
- **Titanium** – E:\$10/kg (200kt/yr, \$30B) – aerospace

STRATEGIC METALS (*T2 via refinery; Earth-return viable*)

- **Cobalt** – E:\$30/kg (170kt/yr, \$5B) – batteries, superalloys
- **Rare earths** – E:\$500/kg (300kt/yr, \$15B) – electronics, magnets
- **Platinum group** – \$30k/kg (500t/yr, \$15B) – catalysts, electronics

VOLATILES (*T4 via refinery/chemical plant*)

- **Ammonia** – E:\$0.5/kg (150Mt/yr, \$75B) – nitrogen source, propellant
- **Methane** – E:\$0.2/kg (4Tm³/yr, \$200B) – fuel, feedstock
- **Nitrogen** – E:\$0.1/kg (150Mt/yr, \$15B) – life support, propellant
- **CO₂** – E:\$0.05/kg (230Mt/yr, \$10B) – life support, propellant
- **Propellant** – E:\$5/kg – processed from water or methane

ORGANICS & CHEMICALS (*T4 via chemical plant*)

- **Carbon/Organics** – E:\$1/kg (3Mt/yr, \$20B) – plastics, composites, fuel
- **Sulfur** – E:\$0.1/kg (80Mt/yr, \$8B) – industrial chemical, batteries
- **Salts** – E:\$0.5/kg (300Mt/yr, \$15B) – electrolytes, processes
- **Silicon** – E:\$2/kg (8Mt/yr, \$15B) – solar cells, electronics.

TETHER & HIGH-STRENGTH MATERIALS (*T3–T5*)

See *Physics Handbook: Tethers* for v_c formula and mass ratio calculations.

Material	σ (GPa)	ρ (kg/m ³)	v_c (km/s)	E\$/kg	Tier	Notes
Nylon	0.08	1,140	0.27	\$3	T4	Easy to work, poor space performance
Steel (high-strength)	2.0	7,800	0.51	\$1	T2	Heavy; good for ground
Aluminum (7075)	0.5	2,800	0.42	\$5	T2	Light but weak for tethers
Titanium (Ti-6Al-4V)	1.0	4,400	0.48	\$50	T3	Excellent fatigue life
Kevlar 49	3.6	1,440	1.58	\$30	T4	Aramid fiber; standard space tether
Spectra/Dyneema 3.5		970	1.90	\$50	T4	UHMWPE; weak above 100°C
Vectran	3.2	1,400	1.51	\$80	T4	Good thermal stability

Material	σ (GPa)	ρ (kg/m ³)	v_c (km/s)	E\$/kg	Tier	Notes
Zylon (PBO)	5.8	1,560	1.93	\$200	T5	Strongest fiber available; UV-sensitive
Carbon fiber (T1000)	6.4	1,800	1.89	\$100	T5	Stiff; requires matrix
M5 fiber	5.8	1,700	1.85	\$500	T5	Research; heat/UV resistant
Carbon nanotube	60+	1,300	6.8+	\$100k	—	Requires break-through; D:\$10B, P:0.6

EXOTIC

- **Helium-3** — \$1M/kg (0 Earth supply; lunar extraction required) — fusion fuel; 20 ppb in lunar regolith

ADDITIONAL MATERIALS *(Referenced in recipes but not in main Materials table)*

- **Copper** — E:\$8/kg — electrical wiring, conductors. *Referenced in recipes as “Cu”.*
- **Lithium** — E:\$50/kg — batteries, lightweight alloys
- **Fabric** — E:\$5/kg — textiles, parachutes, MLI blankets. *Generic term; includes nylon, Kevlar, etc.*

T5 PRECISION COMPONENTS *(Required for recipes marked (T5); must be produced in Precision Fab or purchased from Earth)*

Prices are Earth prices. In space, add launch cost to destination. These are finished components, not raw materials.

- **Wiring (T5)** — E:\$10k/kg — space-rated electrical wiring, connectors, insulation
- **Electronics (T5)** — E:\$50k/kg — circuit boards, processors, sensors, controllers
- **Tooling (T5)** — E:\$20k/kg — precision machine tools, cutting heads, calibration equipment
- **Motors (T5)** — E:\$30k/kg — precision motors, actuators, servos
- **Pumps (T5)** — E:\$25k/kg — precision pumps, compressors, fluid handling
- **Controls (T5)** — E:\$40k/kg — control systems, valves, regulators
- **Mechanisms (T5)** — E:\$35k/kg — precision mechanisms, gears, bearings
- **Membranes (T5)** — E:\$15k/kg — fuel cell membranes, filters, separators
- **Insulation (T5)** — E:\$5k/kg — space-rated electrical insulation, thermal barriers
- **Optics (T5)** — E:\$100k/kg — lenses, mirrors, optical systems
- **Reactors** — E:\$100k/kg — chemical reactors, process vessels (T4, but high precision)
- **Seals** — E:\$2k/kg — gaskets, O-rings, hermetic seals (T3–T4, but space-rated)

Generic pricing for T5 components not listed: \$30k/kg (negotiate based on complexity).

PRICING GENERIC TERMS

When recipes use generic terms, choose appropriate material and use its price:

- **“metal”** — Use cheapest appropriate structural metal (typically Aluminum \$2/kg for lightweight, Iron \$0.1/kg for heavy structures)
- **“chemicals”** — Use appropriate chemical from Materials section (ammonia \$0.5/kg, methane \$0.2/kg, etc.)
- **“fabric”** — Use \$5/kg (generic textile) or specific fabric price if known

3.7 VOLUME DISCOUNTS & MANUFACTURING

3.7.1 Buying vs. Building

Buying: Pay list price. Lead time from N/yr table (see READING THIS CATALOG). Flags (Licensed, Captive, Single-source) may restrict access.

Outsourced AIT: When you buy parts and do **not** manufacture the vehicle in your own factory, you may purchase an **AIT campaign** (integrate, test, launch prep) from a vendor. Negotiate flat cost — typical: cubesat LEO bus ~\$100k; propulsive kick / smallsat-class stack ~\$150k; heavier or hosted integrate, argue up. AIT is **not** Building (no D_{dev} , no volume-discount prototype multiplier, no factory tonnes on your books). Founder or assigned Ground still run the program (design authority, ICDs, accept/reject); Manufacturing or Flight specialty -10% time applies to that program calendar when relevant.

Building: Three cases — (1) **catalog item, no D; P** → pay D_{dev} below, then manufacture; (2) **device with D; P** → research breakthrough first (Making Stuff / Chance), then D_{dev} and manufacture if you want your own line; (3) **new-to-the-world item** → negotiate D and P, research, then set list price and build.

3.7.2 Development Cost

To independently produce a catalog item that does not list D; P:

$$D_{\text{dev}} = \text{Tier multiplier} \times \text{list price}$$

Tier	Dev Multiplier
T1–T2	2×
T3	3×
T4	5×
T5	10×

Development time & cost: Pay D_{dev} (program spend) **and** lab rent or ops for the duration. Route the program spend through matching lab bandwidth you own or rent (Making Stuff); wall-clock = $D_{\text{dev}} \div \text{bandwidth}$, with qualification folded into a **minimum 1 year** total (not stacked on top). Tiny programs only: solo-CEO floor **\$1M/year** with no dedicated lab.

Example: Starlink-class Hall thruster, T5 list \$0.2M → $D_{\text{dev}} = \$2M$ + one year of lab rent/ops at whatever bandwidth you allocate; at \$10M/year bandwidth the burn is weeks but the clock is still 1 year.

3.7.3 Volume Discounts

Applies to hardware you build: custom rockets, components, factory output. Identical designs only; changes reset counter. *Pre-built launch services (Falcon 9, etc.) are already at provider's mature pricing.*

Units Built	Multiplier
1–5	5.0× (prototype)
6–20	3.5× (learning curve)
21–100	2.5× (established)
101–200	1.5× (mass production)
201+	1.0× (SpaceX-level)

Compare your units built to the item's N/yr to gauge market position. If the world makes 50/yr and you've built 200+, you're a major producer at mature pricing. If N/yr is 5000 and you've built 20, you're a niche player still on the learning curve.

Per-unit build cost: (Materials + Engineering) $\times$ 1.2 $\times$ Multiplier

Materials: Sum of (material mass $\times$ price/kg) from recipe.

Engineering: Base engineering/design/testing cost that scales with tier: - **T1–T2:** \$10k/t — Simple structures, basic fabrication - **T3:** \$100k/t — Mechanical systems, moving parts, seals - **T4:** \$500k/t — Chemical processing, complex systems - **T5:** \$2M/t — Precision electronics, optics, engines

The 1.2 $\times$ covers integration and final assembly. *This is what it costs to build. Operating costs (ops people, ground contact, consumables, insurance) are separate; see Operations.*

3.7.4 Improving Existing Tech

To improve an existing item's specs by X%:

$$D_{\text{improve}} = D_{\text{dev}} \times \left(\frac{X}{25} \right)^2$$

P: negotiate based on physics. 10% improvement costs 16% of D_{dev} . 25% costs 100%. 50% costs 400%. For breakthrough improvements (new materials, new physics), use D; P research system.

3.8 MARKETS & COMPETITION

Price your offer against Shop alternatives (or real analogues) and argue the sale with the other player. New goods or markets: propose use, customers, and price; adjudicate; research if you need facts that aren't in the book yet.

DEMAND

Total market at price P :

$$D = N \times \left(\frac{A}{P} \right)^E$$

- **N** — Baseline demand. Units/year at the alternative price. Count actual economic activity: missions planned, satellites launching, customers with the need.
- **A** — Alternative price. What the current solution costs. Derive from Shop or real-world equivalent.
- **E** — Elasticity. How price-sensitive is demand?

E	Meaning	Examples
0	Inelastic	Life support, abort systems
0.5	Low elasticity	Communications, propellant
1.0	Unit elastic	Most services
2.0+	Elastic	Tourism, luxury

Deriving N, A, and E:

- **N** (baseline demand) — Use **N/yr** from the item's catalog entry (Propulsion, Equipment). For materials, use the annual volume in parentheses. For novel products not in the catalog, count real activity or negotiate.

- **A** (alternative price) — Use the item's **list price** from the catalog. If competing against a different solution (e.g., your propellant depot vs. launching from Earth), use the alternative's cost.
- **E** (elasticity) — Use the **E by category** tables: Material Elasticity (Materials section) or Equipment Market Reference (Equipment section). For propulsion, E is listed per engine class.

Challenge claims with evidence. If neither side has evidence, negotiate a range and roll: $\text{result} = \text{min} + (\text{max} - \text{min}) \times \text{d}100/100$.

- **Earth markets** — Annual demand in Materials section.
- **Space markets** — Demand = active operations' consumption. No operations = no market. As players build infrastructure, N/yr for space-market items grows.
- **New markets** — Emerge only from player actions.

MARKET SHARE (Multiple Suppliers)

When 2+ suppliers compete, customers flow to the cheapest **reliable** option, up to that supplier's **capacity**.

Step 1: Rank suppliers by price, adjusted for reputation (see Reputation table below).

Step 2: Allocate demand: 1. Cheapest adjusted price captures demand up to their stated capacity 2. Remaining demand flows to next-cheapest 3. Continue until demand exhausted or suppliers exhausted 4. If adjusted prices are equal (or within ~5%), split demand proportional to capacity

Step 3: If you're priced out or capacity-constrained, you can: drop price (takes effect next quarter), expand capacity, or exit market.

Example: Lunar propellant. N = 100t/yr at A = \$50k/t. E = 0.5. - Alice: \$30k/t, Trusted (0.9×) → adjusted \$27k, capacity 60t/yr - Bob: \$35k/t, Unproven (1.3×) → adjusted \$45.5k, capacity 100t/yr

At Alice's \$27k adjusted price, demand = $100 \times (50/27)^{0.5} = 136\text{t}$. Alice sells 60t (capacity max). Remaining customers face Bob's \$45.5k. At that price, total market demand = $100 \times (50/45.5)^{0.5} = 105\text{t}$. Alice already served 60t, leaving 45t for Bob. Bob captures 45t/yr of sales.

COMPETITION ENTRY (NPCs)

High profits attract capital. When you're making >30% margins, competitors smell blood.

Entry timing: Argue based on barriers. How hard is it to replicate what you're doing?

How we played it: they take the same amount of time you took, from demonstration of profitability.

- **Low barriers** (Earth manufacturing, services) — 1–3 years
- **Medium barriers** (launch vehicles, complex tech) — 3–5 years
- **High barriers** (space infrastructure, novel R&D) — 5–10+ years

If you're making \$50M/yr profit and entry costs \$100M, expect faster entry (~2yr) than if profit is \$5M/yr (10yr+). Capital flows to returns.

Entry capacity: When NPCs arrive, they bring capacity equal to **25–50% of current market demand**. More if margins are exceptional (>50%), less if marginal (<30%). Negotiate or roll d100: capacity % = 25 + (d100/4).

NPC behavior: + **Pricing** — NPCs target 15–20% margins. They price at their cost + 15–20%. + **Cost structure** — Assume NPCs have similar costs to established players (your cost after learning curve, or Shop baseline). + **Reputation** — Start as Unproven (×1.3 adjustment). After 3 years, become Established (×1.0). + **Failures** — NPCs don't catastrophically fail. They're competent but not exceptional. Ignore individual NPC mission outcomes unless a player specifically targets them (sabotage, lawsuits, etc.).

Ongoing entry: If margins stay >25% after first NPC entry, another wave arrives in 2–3 years. Continues until margins compress to 15–20% equilibrium.

NPCs as customers: New competitors need infrastructure.

- **Manufacturing** — Need factories (can they build? buy? rent from you?)
- **Propellant/Materials** — Buy from you or compete with you for mining sites
- **Transport** — Need launch, cargo services
- **Power** — Need electricity, solar arrays, reactors
- **Operations** — Need crew, ground control

When NPCs enter, calculate their input needs (same recipes/ratios you use). They create new demand for your upstream services. You can profit from competitors if you control critical infrastructure.

Blocking competition:

- **Long-term contracts** — Lock up customers before competitors arrive. If you have 5-year contracts covering 80% of market, NPCs only compete for 20%.
- **Exclusive resources** — Own the only ice mine on the Moon? NPCs must buy water from you or fund their own mine (adds to their capital requirement).
- **Predatory pricing** — Drop price to cost (0% margin) to signal “unprofitable market.” Deters but doesn’t prevent entry. You take losses while deterring.
- **Capacity flooding** — Build excess capacity to credibly threaten price war. Idle capacity still requires ops people + contact if monitored.
- **Technical moats** — If your advantage is proprietary tech, NPCs can’t replicate without R&D investment. Argue case for extended delay.

Player entry: No delay. You can build capacity and compete in any market immediately.

CONTRACTS

Negotiate what works. Common patterns:

- **Spot sales** — Current price, subject to competition. Flexible but volatile.
- **Fixed contracts** — Lock price and volume for 1–5 years. Buyer gets price certainty; you get committed revenue. Penalties if either party defaults (negotiate severity).
- **Cost-plus services** — Customer pays your cost + margin. Typical 20–50% margin; higher with reputation, lower with competition.
 - *Cost basis:* Launch, crew (\$500k/yr), consumables, spares, insurance (if carried)
- **Repair services** — Customer ceiling = replacement cost. Your floor = spares + crew time + risk.

REPUTATION

Level	Missions	Price Adjustment
Unproven	0–2	×1.3 (customers demand discount)
Established	3–5	×1.0 (market rate)
Trusted	6–10	×0.9 (premium accepted)
Leader	11+	×0.85 (premium accepted)

Price Adjustment: how customers perceive your quoted price when comparing suppliers (see Market Share). Unproven players can charge whatever they want, but customers treat their price as 30% higher — winning competitive bids means undercutting. Monopoly pricing constrained by demand elasticity, not reputation. Mission count sets your maximum level. Failures drop you below max: major -2, partial -1. Each success restores one level (up to max).

4. Making Stuff

Research labs and factories. Spin up, run, spin down.

4.1 FACILITIES

Research labs turn money into breakthroughs. **Bandwidth** is how much research investment (\$) the lab can process per year. Research time = difficulty (D) ÷ bandwidth (minimum one month).

Factories turn materials into finished items. **Bandwidth** is how much mass (t) the factory can produce per year. Daily output = bandwidth ÷ 365 (minimum one day per run).

Type is a cost multiplier on spin-up and operating cost. Higher Type = more expensive facility.

On-type labs only run their field. A **General Lab** may run any field but pays Type × 1.5 for that flexibility.

4.1.1 Lab Types

Field; power; Type; F → Odds entry

- **Propulsion Lab** — Propulsion; 5 MW per \$1B/year; Type × 2.0; F:0.5 → *Lab failure*
- **Materials Lab** — Materials science; 2 MW per \$1B/year; Type × 1.0; F:0.6 → *Lab failure*
- **Life Support Lab** — Life support; 2 MW per \$1B/year; Type × 1.0; F:0.7 → *Lab failure*
- **Electronics Lab** — Electronics, computing; 3 MW per \$1B/year; Type × 1.5; F:0.8 → *Lab failure*
- **Physics Lab** — Fundamental physics; 10 MW per \$1B/year; Type × 3.0; F:1.0 → *Lab failure*
- **General Lab** — Any field; 3 MW per \$1B/year; Type × 1.5; F:0.6 → *Lab failure*

Lab power = listed MW × (bandwidth / \$1B/year), minimum 100 kW.

4.1.2 Factory Types

Max complexity; power; Type; F → Odds

- **Structural Fab** — T1–T4; 0.1 MW per 100t/year; Type × 1.0; F:0.8 → *Factory failure*
- **Mechanical Shop** — T1–T4; 0.2 MW per 100t/year; Type × 1.5; F:1 → *Factory failure*
- **Chemical Plant** — T1–T4; 0.3 MW per 100t/year; Type × 1.5; F:1.2 → *Factory failure*
- **Precision Fab** — T1–T5; 0.5 MW per 10t/year; Type × 3.0; F:1.5 → *Factory failure*

Factory power = listed scaling × bandwidth.

Complexity tiers: T1 bulk → T2 structural → T3 mechanical → T4 chemical → T5 precision (electronics, optics, engines). Item tier must be ≤ factory max tier. T5 in space: Precision Fab; first five units of a new design are prototypes (×2 F).

4.1.3 Power

Required while the facility is working (not during spin-up). Space must supply it; Earth assumes grid. At continuous load: $\text{kWh/day} = \text{kW} \times 24$.

Insufficient power $\rightarrow$ research/production stops (no partial output). **Operating costs still accrue** unless you idle the facility. After power returns, work may resume in one week.

4.2 RESEARCH

Use this for Shop **D**; **P** breakthroughs and new-to-the-world items. Copying a catalog item with no D; P is Shop D_{dev} (optionally spent through lab bandwidth) – not this section.

D is the investment for 63% success. **F** = $1/D$ for Chance (same USD unit for D and commitment).

Category	D	F (per \$B)	Examples
Incremental	\$50M	20	10% better Isp, lighter tanks
Novel application	\$200M	5	ISRU propellant, space manufacturing
Frontier	\$1B	1	Nuclear thermal, fusion power
Speculative	\$5B+	0.2	Fusion propulsion, antimatter

1. **Lab** – Match field (or General Lab). Allocate bandwidth (several projects OK if total $\leq$ bandwidth).
2. **Spend** – Commit research \$ **and** pay lab ops for the duration. Time = $D \div$ allocated bandwidth (min one month).
3. **Roll** – Chance: Research. If $X \leq$ commitment $\rightarrow$ breakthrough.
4. **Prototype** – First five units built have $\times 2$ failure rate; then specs finalize.

Lab failure: pause (progress kept); same-type lab or repair (Odds). Research \$ not refunded.

Example: D:\$1B at \$5B/year Propulsion Lab $\rightarrow$ 0.2 yr, 25 MW, ops \$200M; commit \$1B and roll.

4.3 SPINNING UP

Pick a type and target bandwidth. Pay cost, wait, then operate at that bandwidth.

Original = bandwidth you grow from. Brand-new facility: the **org floor** from Ground staff (Personnel) – **\$1M/year** and **1 t/year** per Ground person. Expanding: original = current bandwidth.

No converting one type into another. Concurrent spin-ups need one assigned person each (Personnel: Attention) – unlimited parallel without hires is a free lunch.

Time

$$\text{Time (years)} = 0.5 \times \ln\left(\frac{\text{target}}{\text{original}}\right)$$

Cost

$$\text{Cost} = \text{Type} \times 0.1 \times (\text{target} - \text{original}) \times u$$

	Bandwidth unit	u
Labs	\$/year	\$1
Factories	t/year	\$1M per (t/year)

Track **capital** = sum of spin-up costs paid (used for salvage and Odds repair value).

Examples: Materials $\times 1$ to \$1B/year $\rightarrow$ ~ 3.5 yr, $\sim$ \$100M; expand to \$5B $\rightarrow$ ~ 0.8 yr, \$400M. Structural $\times 1$ to 1 kt/year $\rightarrow$ ~ 3.5 yr, $\sim$ \$100M. Propulsion $\times 2$ to \$5B/year $\rightarrow$ ~ 4.3 yr, $\sim$ \$1B.

4.4 RUNNING COSTS

Operating (per year while not idle):

$$\text{Ops} = \text{Type} \times 0.1 \times \text{bandwidth} \times u$$

Same shape as buying that much capacity in one year. Paid whether or not a project is active.

Idle: 5% of operating. Bandwidth preserved. **Restart takes one month** (no capital cost).

If you pay nothing (not even idle), the facility is **abandoned immediately** – gone, no salvage.

4.5 RENTING

$$\text{Rent/year} = 1.3 \times \left(\text{Ops} + \frac{\text{capital}}{5} \right) = 0.156 \times \text{Type} \times B \times u$$

B = rented bandwidth; Type = facility type multiplier; u = \$1 (labs) or \$1M per (t/year) (factories). Second form uses Ops = capital $\approx$ Type $\times$ 0.1 $\times$ B $\times$ u (floor ignored).

Tenant directs work; rent covers ops. Lessor pays repairs and upgrades. Rent due if the project pauses. Pro-rate by B .

Example: Materials $\times 1$, B = \$1B/year $\rightarrow$ \$156M/year.

4.6 SPINNING DOWN

Takes **one month**. Recover **half of capital** (salvage); the rest is lost.

4.7 MANUFACTURING

1. **Tier** – Item tier $\leq$ factory max tier.
2. **Materials** – Shop recipe % are by weight. Basics from ISRU or Earth (Shop: Materials). (T5) inputs from a Precision Fab or Earth (Shop: T5 Precision Components). Generic metal = cheapest fitting structural metal (usually Al).
3. **Cost** – $\Sigma(\text{mass} \times \text{price/kg})$; in space, add launch cost for Earth-sourced mass.
4. **Time** – mass $\div$ (bandwidth/365), minimum one day.
5. **Power** – Per factory type; space must supply it.

Bandwidth caps tonnes/year. Need more → more calendar time, grow the fab, or add another.

Example: 1t solar panel (T3) at 1 kt/year Mechanical Shop → ~\$1.002M materials, 1 day, 2 MW.

5. Personnel

People. Same rules for the founder and everyone you hire.

You start with **one staff** (the founder). If they leave or burn out, hire another — including a new chief if you need one. You play the **company**; bodies are interchangeable under these rules.

5.1 STAFF

Each person has a **posting**, a **specialty**, and costs money while on the roster.

Posting	Cost	Org floor	Attention (spin-up)	Notes
Ground	\$250k/year	+\$1M/year lab, +1 t/year factory	Yes — one spin-up	Programs, labs, fabs
Flight	\$500k/year	—	—	Crew slots; see Flight

Specialists for **T5 operations in space**: 2× Flight cost. Train Ground→Flight specialist: 1 year + \$1M.

Specialty (pick on hire / for the founder):

Specialty	Bonus (while on roster and on matching work)
Generalist	None
Lab	-10% time on lab spin-up or research they run
Flight	-10% time on vehicle/mission programs they run
Manufacturing	-10% time on factory spin-up or production programs
People	Company leave $F \times 0.5$

Onboard: 1 month before they count (floor, attention, or crew slot). **Fire**: 1 month severance; off roster immediately.

5.2 ORG FLOOR

Brand-new facility spin-up original (Making Stuff) from **Ground** staff only:

$$\text{Lab floor} = \$1\text{M/year} \times N_{\text{ground}}$$

$$\text{Factory floor} = 1 \text{ t/year} \times N_{\text{ground}}$$

Solo founder (Ground) → \$1M / 1 t. Expanding a facility still uses current bandwidth as original.

Example: One Ground → \$1B lab ≈ 3.5 years. Two Ground → floor \$2M/year ≈ 3.1 years.

5.3 ATTENTION

Each spin-up needs **one Ground staff** assigned. One person, one spin-up. Want two at once → two Ground people free to assign.

Ground on **ops** (Shop: Operations) spend their attention on standing load ℓ and peaks — not a spin-up. Lights-out standing may share with *idle* lab/factory work only in months with no peak.

5.4 FLIGHT

Mission crew needs (same as former Shop: Crew):

- **Uncrewed** — Avionics only; relay beyond 20 light-minutes
- **Crewed spacecraft** — 1 Flight staff minimum; +1 per 10 t active equipment
- **Station/Base** — 1 Flight staff per 5 t factory/refinery capacity

Remote ops (no one aboard): Ground on ops (standing ℓ + peaks) per Shop: Operations — not separate Flight hires. Contact (antennas / network) is Shop, not a Flight seat.

Rotation: Flight staff in space 1 year max without gravity or major life-support upgrades. Replacement transit counts against mission time. Pro-rate Flight pay by time on mission; minimum **1 month** per person assigned.

5.5 LEAVE & BURNOUT

Same for every staff member (founder included). Chance, $P = 1$; check yearly (and when Pushing for burnout).

Event	F	Effect
Leave	0.2 / year (~5-year tenure)	Off roster after 1 month notice
Burnout	0.1 / year	Off roster immediately; 3 months before they can be hired again (by you or anyone)

Off roster = no pay, no floor, no attention, no crew slot. Replace them like any other gap — hire (1 month onboard).

Push: On an assigned spin-up or crunch program, -20% that work's time; that person's burnout $F \times 2$ for the year.

5.6 RELATIONSHIPS

Argue loyalty, poaching, equity, and politics when it matters. Nuance is free.

6. Physics Handbook

The math of space.

- **Quick Δv** — See Δv Quick Reference table
- **Rocket sizing** — Tsiolkovsky equation (ROCKET PHYSICS)
- **Orbital mechanics** — Vis-viva, Hohmann, rendezvous (TRANSFERS)
- **Surface operations** — Ascent, landing, entry (SURFACE OPERATIONS)
- **Trip times** — Kepler’s third law (ORBITAL DYNAMICS)
- **Lagrange points** — Insertion, halo/Lissajous upkeep, exit (LAGRANGE POINTS)

6.1 REFERENCE

6.1.1 Definitions

- μ (**gravitational parameter**): GM where G = gravitational constant, M = mass of central body. Units: km^3/s^2 . Appears in all orbital mechanics equations. To get mass: $M = \mu/G$ where $G = 6.674 \times 10^{-20} \text{ km}^3/(\text{kg}\cdot\text{s}^2)$.
- Δv (**delta-v**): Change in velocity. The total velocity budget needed for any maneuver; independent of how or when thrust is applied. Δv ’s in the same reference frame add directly. When switching frames (e.g., planetary orbit $\rightarrow$ heliocentric), compute velocities in the new frame and find the difference.
- v_e (**exhaust velocity**): Speed at which rocket exhaust leaves the engine. $v_e = \text{Isp} \times g_0$ where $g_0 = 9.81 \text{ m/s}^2$.
- H (**scale height**): Atmosphere density decreases exponentially with altitude: $\rho(h) = \rho_0 e^{-h/H}$. Larger H means density falls off more slowly.
- a (**semi-major axis**): Half the longest diameter of an ellipse. For a circular orbit, $a = r$. For hyperbolic escape trajectories, a is negative.
- ε (**specific orbital energy**): Orbital energy per kg mass: $\varepsilon = KE/M + PE/M = v^2/2 - \mu/r = -\mu/(2a)$. Conserved along any unperturbed orbit; change in energy equals work done by thrust.
- r_p (**periapsis**) / r_a (**apoapsis**): Closest / farthest point in orbit. (Earth: perigee/apogee. Sun: perihelion/aphelion.)
- e (**eccentricity**): Shape of orbit. $e = 0$ (circle), $0 < e < 1$ (ellipse), $e = 1$ (parabola), $e > 1$ (hyperbola).
- ν (**true anomaly**): Angle from periapsis to current position in orbit (0° to 360°).
- i (**inclination**): Tilt of orbit relative to reference plane (degrees).
- Ω (**RAAN**): Right ascension of ascending node — where the orbit crosses the reference plane northward (0° to 360°).
- ω (**argument of periapsis**): Angle from ascending node to periapsis, measured in the orbital plane (0° to 360°).
- **Orbital elements**: A full orbit is specified by six elements: $a, e, i, \Omega, \omega, \nu$. First two define shape, next two define orientation, last two define position.
- v_∞ (**hyperbolic excess velocity**): Speed “at infinity” after escaping a gravity well. $a = -\mu/v_\infty^2$. Also: difference between your heliocentric velocity and the planet’s orbital velocity.
- r_{SOI} (**sphere of influence**): Radius within which a body’s gravity dominates over the parent body: $r_{\text{SOI}} = a(m/M)^{2/5}$ where a = orbital distance, m = body mass, M = parent mass. Inside SOI, use μ_{body} ; outside, use μ_{parent} .
- g (**surface gravity**): $g = \mu \times 1000/r^2 \text{ (m/s}^2\text{)}$, where r = body radius (km).
- **Lagrange (libration) points**: Five balances in the frame corotating with the primary–secondary line: **L1–L3** collinear; **L4–L5** equilateral. Not surfaces to land on. **Halo / Lissajous**: periodic or quasi-periodic loop **around** a collinear point (empty center). Exact three-body paths need numerics; **Lagrange Points** gives table values for play.

6.1.2 Abbreviations

- **LEO:** Low Earth Orbit (~200–2000 km altitude)
- **GEO:** Geostationary Orbit (~35,786 km, synchronous with Earth rotation)
- **Isp:** Specific impulse (seconds); higher = better fuel efficiency
- **TWR:** Thrust-to-Weight Ratio
- **c (speed of light):** 299,792 km/s $\approx$ 1 AU / 8.3 min
- **EML1 / EML2:** Earth–Moon L1 / L2
- **SEL1 / SEL2:** Sun–Earth L1 / L2

6.2 ORBITAL DYNAMICS

6.2.1 Vis-Viva (Speed Anywhere in Any Orbit)

Total orbital energy is conserved: $v^2/2 - \mu/r = -\mu/2a$. Where r = current distance from central body, a = semi-major axis. Solve for v :

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

Or solve for a :

$$a = \frac{\mu}{2\mu/r - v^2}$$

Works for all orbit types:

- **Circular** ($a = r$): Constant velocity $v = \sqrt{\mu/r}$, no eccentricity
- **Elliptical** ($a > 0$): Bound orbit with eccentricity $0 < e < 1$
- **Hyperbolic** ($a < 0$, use $a = -\mu/v_\infty^2$): Unbound escape trajectory with excess velocity
 - $v = \sqrt{v_\infty^2 + 2\mu/r}$
 - v is the velocity at any radius r when departing/arriving with excess velocity v_∞ . As $r \rightarrow \infty$, $v \rightarrow v_\infty$.
- **Parabolic** ($v_\infty = 0$): Just barely escapes, $a \rightarrow \infty$

Oberth effect: A burn at low r (deep in a gravity well) produces a larger change in orbital energy than the same burn at high r . Kinetic energy is $\frac{1}{2}v^2$, and adding Δv to a larger v yields more energy: $(v + \Delta v)^2 - v^2 = 2v\Delta v + \Delta v^2$. Always burn at periapsis when possible.

6.2.2 Kepler's Third Law (Orbital Period)

$$T = 2\pi \sqrt{\frac{a^3}{\mu}}$$

Time to travel from periapsis to apoapsis (half-orbit): $t_{\text{half}} = \pi \sqrt{a^3/\mu}$

Kepler's second law: Equal areas swept in equal times. Near periapsis (fast, close), you cover more angle per time; near apoapsis (slow, far), less angle per time.

This surprisingly simple law gives us a method for projecting any orbit forward in time, analytically.

To trace an orbit, you should first figure out the angle of your orbit around what it is orbiting (true anomaly, see Definitions).

Finding True Anomaly from Distance

A central problem: you know your distance r from the central body, but where are you in the orbit? Use this to find true anomaly ν .

Distance r at true anomaly ν :

$$r = \frac{a(1 - e^2)}{1 + e \cos(\nu)}$$

Solve for ν given r :

$$\cos(\nu) = \frac{a(1 - e^2) - r}{er}$$

Example: Earth orbit ($a = 7000$ km, $e = 0.1$) at $r = 7500$ km. $\cos(\nu) = (7000 \times 0.99 - 7500)/(0.1 \times 7500) = -0.714$, so $\nu = 135^\circ$ or 225° (check which side of periapsis).

Time-of-Flight for Arbitrary Arcs

Once you know where you are (ν), convert to time. Or work backwards: given time since periapsis, find where you'll be.

Circular ($e = 0$): $t = (\theta/2\pi) \times T$ where θ = angle in radians

Elliptical ($0 < e < 1$): Time from periapsis to true anomaly ν

- Convert $\nu \rightarrow E$: $\cos(E) = (e + \cos(\nu))/(1 + e \cos(\nu))$
- Compute $M = E - e \sin(E)$ (Kepler's equation)
- Time: $t = (M/2\pi) \times T$
- Between two points: calculate time to each from periapsis, subtract

Hyperbolic ($e > 1$): Time from periapsis to true anomaly ν

- Convert $\nu \rightarrow H$: $\cosh(H) = (e + \cos(\nu))/(1 + e \cos(\nu))$
- Compute $M = e \sinh(H) - H$ (hyperbolic Kepler's equation)
- Time: $t = (M/2\pi) \times T$ where T uses $|a|$ (absolute value of semi-major axis; a is negative for hyperbolas)
- Between two points: calculate time to each from periapsis, subtract

6.3 TRANSFERS

6.3.1 How Transfers Work: Tangent Ellipses

To move from one orbit to another, you use a **transfer ellipse** that touches both orbits. The key insight: at the touch points, the transfer orbit is **tangent** to the original orbits—same radius, different velocity.

The process:

1. At the departure point: burn to change velocity from your current orbit to the transfer ellipse
2. Coast along the transfer ellipse
3. At the arrival point: burn to match the target orbit

Why tangent? If the orbits cross at an angle, you waste Δv fighting the velocity mismatch. Tangent burns are pure energy changes—most efficient.

6.3.2 Hohmann Transfer (circular to circular)

The **Hohmann transfer** is the minimum-energy two-burn transfer between circular orbits. The transfer ellipse has periapsis at r_1 and apoapsis at r_2 , so $a_t = (r_1 + r_2)/2$. In this situation, vis-viva simplifies considerably. Because of how often Hohmann transfers are used, let's cover that.

Procedure:

1. Calculate $v_r = \sqrt{\mu/a_t}$, a reference velocity.
2. **Burn 1** (at r_1): Accelerate from circular velocity to transfer orbit. $v_{t,1} = v_r \sqrt{r_2/r_1}$ and $\Delta v_1 = v_{t,1} - \sqrt{\mu/r_1}$.
3. **Coast** for half the transfer period: $t = \pi \sqrt{a_t^3/\mu}$
4. **Burn 2** (at r_2): Circularize. $v_{t,2} = v_r \sqrt{r_1/r_2}$ and $\Delta v_2 = \sqrt{\mu/r_2} - v_{t,2}$.

Note that $v_{t,1}v_{t,2} = v_r^2$, so you can also calculate $v_{t,2} = v_r^2/v_{t,1}$.

Total: $\Delta v = \Delta v_1 + \Delta v_2$

Example: LEO (200 km) to GEO (35,786 km): $\Delta v = 3.9$ km/s, transfer time = 5.2 hours.

Bi-elliptic transfer: For large radius ratios ($r_2/r_1 > 11.94$), a three-burn transfer through a very high intermediate orbit beats Hohmann. Burn to high apoapsis, circularize, then transfer down. Slower but cheaper in Δv .

6.3.3 Phasing Orbits (Rendezvous)

To catch up to a target in the same orbit: transfer to a temporary orbit with a different period, accumulate angular offset, then return.

To catch a target $\Delta\theta$ ahead: Drop to a lower (faster) orbit. Required period: $T_{\text{phase}} = T(1 - \Delta\theta/(2\pi n))$ where n = number of orbits to complete the phase. Compute a_{phase} from Kepler's third law. Two small Hohmann burns: one to enter, one to return.

To fall behind: Raise to a higher (slower) orbit instead.

Example: Target is 30° ahead in a 400 km circular orbit ($T = 92.4$ min). Phase in 1 orbit: $T_{\text{phase}} = 84.7$ min, $a_{\text{phase}} = 6,510$ km (alt ≈ 130 km). Total $\Delta v \approx 0.2$ km/s.

6.3.4 Plane Change

Orbital inclination i is the tilt of your orbit's plane relative to a reference plane (usually the equator for Earth orbits, or the ecliptic for interplanetary). $i = 0^\circ$ means equatorial; $i = 90^\circ$ means polar.

To change inclination by Δi at velocity v :

$$\Delta v_{\text{plane}} = 2v \sin(\Delta i/2)$$

Key points:

- Plane changes are expensive—they scale with velocity, not just the angle. A 90° plane change costs $1.41v$ (more than your orbital speed!).
- Do plane changes at **apoapsis** where v is lowest. This minimizes Δv cost.
- Combine with other maneuvers: do the plane change during the second burn of a Hohmann transfer (at apoapsis).
- Small plane changes are much cheaper: $\sin(\Delta i/2) \approx \Delta i/2$ for small angles, so $\Delta v \approx v \times \Delta i$ (in radians).

Example: 10° plane change at LEO ($v = 7.8$ km/s) costs 1.36 km/s. At GEO ($v = 3.1$ km/s), same 10° change costs only 0.54 km/s—do it there instead.

6.3.5 Orbital Precession (J_2)

Earth's oblateness causes the orbital plane to rotate around Earth's axis. Nodal precession rate:

$$\dot{\Omega} \approx -\frac{3}{2} \frac{J_2 R_\oplus^2 n}{a^2 (1 - e^2)^2} \cos i$$

$J_2 \approx 1.08 \times 10^{-3}$, $R_\oplus$ = Earth radius, a = semi-major axis, e = eccentricity, i = inclination, n = mean motion. For circular orbits: $\dot{\Omega} \propto \cos i / a^{7/2}$.

Direction: Prograde ($i < 90^\circ$) → plane precesses westward. Retrograde ($i > 90^\circ$) → eastward. Polar ($i = 90^\circ$) → no precession ($\cos 90^\circ = 0$).

Sun-synchronous: Choose a and i so $\dot{\Omega} \approx 0.986^\circ/\text{day}$ (Earth's orbital rate). At 600–800 km altitude, $i \approx 98^\circ$. Same lighting every pass—used for EO, recon. See Astrodata: Earth Orbital Regimes.

6.3.6 Low-Thrust Spiral

For ion engines or solar electric propulsion. Continuous small tangential thrust; orbits remain approximately circular throughout.

Transfer time:

$$t_{\text{spiral}} = \frac{\sqrt{\mu}}{a_t} \left| \frac{1}{\sqrt{r_1}} - \frac{1}{\sqrt{r_2}} \right|$$

Where a_t = tangential acceleration (km/s²). Divide by 86,400 to convert seconds to days.

Total Δv : $\Delta v_{\text{spiral}} = a_t \times t_{\text{spiral}}$ (acceleration × time). Higher than Hohmann for the same endpoints (no Oberth benefit). Add mission margins (see ROCKET PHYSICS).

Number of orbits: $N \approx t_{\text{spiral}} / T_{\text{avg}}$ where T_{avg} uses $a_{\text{avg}} = (r_1 + r_2)/2$.

Example: LEO to GEO with SEP ($a = 0.0001 \text{ km/s}^2$): $\Delta v \approx 4.2 \text{ km/s}$, ~ 180 orbits, ~ 486 days.

6.3.7 Numerical Integration

For trajectories where analytical methods don't work: non-tangential thrust, plane changes mid-transfer, arbitrary thrust profiles, or non-circular departure/arrival orbits.

Equation of motion:

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r} + \mathbf{a}_{\text{thrust}}$$

Procedure:

1. Set initial position $\mathbf{r}_0$ and velocity $\mathbf{v}_0$
2. Define thrust profile: magnitude and direction vs. time
3. Use numerical integrator (RK4, RK45, etc.) to propagate forward
4. Track total Δv : $\Delta v = \int_0^{t_f} |\mathbf{a}_{\text{thrust}}| dt$
5. Stop when target orbit is reached

Verification: Compute specific energy $\varepsilon(t) = |\mathbf{v}|^2/2 - \mu/|\mathbf{r}|$ at each step. For efficient (tangential) transfers, ε should increase monotonically. Large oscillations indicate inefficient or radial thrust.

6.3.8 Escape and Capture

Apply vis-viva (hyperbolic case) in the planetary frame. Burn $\Delta v = |v_{\text{hyp}} - v_{\text{orbit}}|$ where $v_{\text{hyp}} = \sqrt{v_{\infty}^2 + 2\mu/r}$ and $v_{\text{orbit}} = \sqrt{\mu/r}$. Same formula for escape or capture. Burn at periapsis for Oberth benefit.

Example: Escape Earth from 200km LEO with $v_{\infty} = 3$ km/s requires $\Delta v = 3.6$ km/s.

Aerobraking trades fuel for time. Use atmospheric drag instead of a capture burn. For a single pass: $\Delta v_{\text{aero}} \approx (v_{\text{approach}} \times \rho_0 \times H/BC) \times e^{-h_p/H}$, where $BC = m/(C_d \times A)$ is ballistic coefficient ($C_d \approx 2.2$ for blunt bodies), ρ_0 = surface density, H = scale height, h_p = periapsis altitude (all SI units). Multiple passes may be needed.

6.3.9 Gravity Assists

Fly past a planet on a hyperbolic trajectory. Your velocity relative to the planet stays constant, but the planet's motion changes your heliocentric velocity.

Velocity change (heliocentric frame): $\Delta v_{\text{assist}} = 2v_{\infty} \sin(\delta/2)$, where δ = turn angle.

Turn angle: $\sin(\delta/2) = 1/(1 + r_p v_{\infty}^2 / \mu_{\text{planet}})$, where r_p = closest approach. Lower periapsis = sharper turn = more Δv .

Direction: Pass behind planet (in its orbit) to speed up. Pass in front to slow down.

Example: Jupiter assist can add 10+ km/s — essential for outer solar system missions.

6.3.10 Launch Windows

When to launch for interplanetary transfers.

Phase angle at departure: The target planet must be ahead (outer) or behind (inner) by the right amount so you arrive when it does.

$$\theta = 180^\circ - \frac{180^\circ \times T_{\text{transfer}}}{T_{\text{target}}}$$

Where T_{transfer} = transfer time (from Kepler), T_{target} = target planet's orbital period. Positive θ = target ahead of Earth; negative = behind.

Window frequency (synodic period): Time between consecutive launch opportunities.

$$T_{\text{synod}} = \frac{1}{|1/T_1 - 1/T_2|}$$

Example: Earth-Mars. Transfer = 260 days. Mars period = 687 days. Phase angle = $180^\circ - 180^\circ \times 260/687 = 112^\circ$. Mars must be 112° ahead of Earth. Windows occur every 780 days (2.1 years).

Days until window: From current positions (Astrodata), calculate angular separation. Rate of closure = $360^\circ/T_{\text{synod}}$. Days = (required angle - current angle) / rate, mod T_{synod} .

Distance between bodies: For two bodies orbiting the same central body at distances R_1 and R_2 with angular separation θ : Distance = $\sqrt{R_1^2 + R_2^2 - 2R_1R_2 \cos(\theta)}$. Convert degrees to radians: $\theta_{\text{rad}} = \theta_{\text{deg}} \times \pi/180$. *Example: Earth-Mars at opposition (180°) = 2.52 AU; at conjunction (0°) = 0.52 AU.*

6.3.11 Station-Keeping

Orbits decay from drag, solar pressure, and gravitational perturbations. Budgets per year:

Regime	$\Delta v/\text{year}$	Primary cause
LEO (200–400 km)	50–100 m/s	Atmospheric drag
LEO (600+ km)	< 5 m/s	Minimal drag
GEO	50 m/s	Solar/lunar perturbations, N-S + E-W
Sun-synchronous	5–15 m/s	Drag + J_2 maintenance
Lunar orbit (low)	50–200 m/s	Mascon perturbations
Lagrange points (L1–L3)	15 m/s	Halo/Lissajous upkeep; see Lagrange Points

For drag-dominated orbits: orbital lifetime without station-keeping $\approx (h \times BC)/(H \times \rho_0 \times v_{\text{orbit}} \times 86400)$ days, where BC = ballistic coefficient, h = altitude, H = scale height, ρ_0 = density at altitude.

6.3.12 Lagrange Points

*No Keplerian closed form for the restricted three-body problem. At the table: pay normal two-body legs first (Quick Reference, **Launch Windows, Escape and Capture**), then a single **final trim** into a halo/Lissajous (L1–L3) or libration (L4–L5), yearly **SK**, and the same trim to **exit**.*

Legs: (1) Approach to the secondary’s SOI or heliocentric leg as usual. (2) Capture if needed. (3) **Final trim** over a few days (several small burns).

Final trim Δv — not inclusive of LEO or planetary escape; **from** low circular lunar orbit (Earth–Moon), heliocentric approach already at the L neighborhood (Sun–Earth), or low Jovian orbit (Sun–Jupiter).

System	L1 / L2	L3	L4 / L5
Earth–Moon	0.75 km/s	0.65 km/s	0.25 km/s
Sun–Earth	0.40 km/s	0.35 km/s	0.10 km/s
Sun–Jupiter	0.12 km/s	0.12 km/s	0.05 km/s

Station-keeping: L1–L3: **15 m/s/yr**. L4–L5: **5 m/s/yr** if holding a tight station; **0 m/s/yr** orbit trim if you only librate and accept loose geometry (attitude still uses wheels / usual ACS).

If you skip SK: L1–L3 — after **8 weeks** without trim, leave the controlled halo; **recovery** = **0.40 km/s** once, then resume 15 m/s/yr. L4–L5 — after **12 months** without the 5 m/s/yr hold, apply **0.12 km/s** to realign or accept large libration vs. targets.

Exit: Same final trim as insertion for that point. Optional low-energy path: **-0.20 km/s** on trim and **+60 days** flight time (manifold / flyby routing).

Checks: Budget paid $\rightarrow$ no roll. Failed nav/control on a trim: **+0.10 km/s** on that leg **or 7 days** delay for a new solution.

6.4 SURFACE OPERATIONS

6.4.1 Surface $\leftrightarrow$ Orbit

Ascent

$$\Delta v_{\text{ascent}} = v_{\text{orbit}} + \text{Gravity Loss} + \text{Drag Loss}$$

Where v_{orbit} comes from vis-viva (circular case) at target altitude.

Gravity loss: Gravity opposes thrust during ascent. Higher TWR = shorter burn = less loss.

$$\text{Gravity Loss} = g_{\text{avg}} \times t_{\text{burn}}$$

$$g_{\text{avg}} = \frac{1}{2} \left(\frac{\mu}{R^2} + \frac{\mu}{r^2} \right) (\text{km/s}^2), \quad t_{\text{burn}} = \frac{v_{\text{orbit}}}{(\text{TWR} - 1) \times g_{\text{surface}}}$$

Where R = body radius, r = orbital radius.

Drag loss: Drag Loss $\approx H/100$ (km/s). Empirical ($\pm 50\%$). No atmosphere $\rightarrow$ zero drag loss.

Example: Earth to 200km LEO requires $\Delta v \approx 9.2$ km/s.

Landing

Landing is ascent reversed: $\Delta v_{\text{landing}} = v_{\text{orbit}} + \text{Gravity Loss} - \text{Aerobraking Savings}$. Same gravity loss calculation. For bodies with atmosphere, aerobraking (see TRANSFERS) can eliminate most of the landing burn.

Suicide burn (airless bodies): Begin braking burn at altitude $h_{\text{start}} = v^2 / (2(a_{\text{thrust}} - g))$ where v = current velocity, $a_{\text{thrust}} = F/m$. Starting later saves gravity losses but leaves no margin.

Example: Moon from 100 km orbit: $\Delta v \approx 1.7$ km/s (no atmosphere).

6.4.2 Atmospheric Entry and Burnup

What burns up: Unprotected objects entering atmosphere at high velocity. Typical cases: spent stages, debris, satellites, or payloads without heat shields.

Quick decision (Earth):

Entry Velocity	Outcome
< 7 km/s	Survives (LEO deorbit)
7–11 km/s	Dense objects survive; lightweight burns up
> 11 km/s	Burns up (unless heat shield)

Ballistic coefficient: $BC = m / (C_d \times A)$ where m = mass (kg), A = cross-sectional area (m^2), $C_d \approx 2.2$ for blunt bodies.

- **High BC** ($> 1000 \text{ kg/m}^2$): Dense stages, metal tanks $\rightarrow$ survive up to ~ 11 km/s
- **Low BC** ($< 100 \text{ kg/m}^2$): Lightweight structures, empty tanks $\rightarrow$ burn up above ~ 7 km/s

Heat shields: Required for crewed vehicles or valuable payloads entering > 7.5 km/s. Mass: 15% of entry mass (ablative). See Shop: Heat Shield for cost and specs.

Other atmospheres: Scale velocity thresholds by atmospheric density (from Astrodata). Heating scales with density, so denser atmospheres burn up at lower velocities.

$$v_{\text{burnup}} = v_{\text{earth}} \times \left(\frac{\rho_{\text{earth}}}{\rho_{\text{body}}} \right)^{1/3}$$

Where ρ = surface density (kg/m^3) from Astrodata. Earth: $\rho = 1.2 \text{ kg/m}^3$.

Example: Object entering Venus at 3 km/s burns up (equivalent to ~ 11.5 km/s on Earth). Same object entering Mars at 15 km/s survives (equivalent to ~ 3.8 km/s on Earth).

6.5 ROCKET PHYSICS

6.5.1 Tsiolkovsky Rocket Equation (Fuel for Any Δv)

$$\Delta v = v_e \times \ln \left(\frac{m_{\text{wet}}}{m_{\text{dry}}} \right)$$

Where m_{wet} = initial mass (payload + fuel + structure), m_{dry} = final mass (payload + empty structure).

Rearranged for mass ratio: $m_{\text{wet}}/m_{\text{dry}} = e^{\Delta v/v_e}$

With tanks (s = tank mass ÷ propellant mass; Shop: Tanks, $s = 0.12$), and $R = e^{\Delta v/v_e}$:

$$m_{\text{wet}} = \frac{m_{\text{fixed}} \times R}{1 + s - sR}$$

- m_{fixed} — everything that does not scale with propellant: payload, engines, avionics, stages above.
- Propellant = $(m_{\text{wet}} - m_{\text{fixed}})/(1 + s)$; tanks = $s \times$ propellant.
- **Limit:** a stage needs $R < (1 + s)/s$ (9.33 at $s = 0.12$), so $\Delta v_{\text{max}} = v_e \ln \frac{1+s}{s}$ per stage ($2.23 v_e$).

Example: 1,000 kg fixed, 3 km/s, storable ($v_e = 3.14$ km/s): $R = 2.60$, $m_{\text{wet}} = 2,600/(1.12 - 0.312) = 3,218$ kg; propellant 1,981 kg; tanks 238 kg.

6.5.2 Mission Margins (Delta-v Overhead)

Base margin: +5% per independent propulsion system (mandatory). Covers residuals, navigation errors, contingencies. Applies to in-space Δv ; ascent budgets (e.g. 9.2 km/s to LEO) already include gravity and drag losses and take no extra margin.

Additional margin reduces F: Each +5% above base multiplies propulsion F values by 0.9:

Example: 2-stage mission, 5.7 km/s required. Base +10% (2 systems), additional +10% = 20% total. Size for 6.84 km/s. F multiplier: $\times 0.81$.

6.5.3 Multi-Stage Design

Each stage uses Tsiolkovsky independently. Stages drop off when empty. Work backward from payload:

$$\Delta v_N = v_e \times \ln \left(\frac{m_{\text{payload}} + m_{\text{fuel},N} + m_{\text{stages below}}}{m_{\text{payload}} + m_{\text{stages below}}} \right)$$

6.6 Δv QUICK REFERENCE

Hohmann transfers from Earth (200 km LEO). All values approximate; actual Δv depends on launch window and trajectory.

Destination	Δv from LEO	Transfer Time	Notes
Earth Orbit			
LEO → GEO	3.9 km/s	5 hours	
LEO → Moon orbit	3.1 km/s	3 days	

Destination	Δv from LEO	Transfer Time	Notes
LEO $\rightarrow$ Moon surface	5.9 km/s	3 days	includes landing
LEO $\rightarrow$ Earth escape	3.2 km/s	—	$v_\infty = 0$
Inner Planets			
Earth $\rightarrow$ Venus	3.5 km/s	150 days	+ 3.3 km/s capture
Earth $\rightarrow$ Mars	3.6 km/s	260 days	+ 2.1 km/s capture
Asteroid Belt			
Earth $\rightarrow$ Vesta (2.4 AU)	5.5 km/s	1.1 years	+ 2.3 km/s capture
Earth $\rightarrow$ Ceres (2.8 AU)	4.9 km/s	1.3 years	+ 4.6 km/s capture
Earth $\rightarrow$ belt edge (3.3 AU)	4.5 km/s	1.6 years	flyby only
Outer Planets			
Earth $\rightarrow$ Jupiter	6.3 km/s	2.7 years	+ 17 km/s capture (use aerobrake or assist)
Earth $\rightarrow$ Saturn	7.3 km/s	6 years	+ 10 km/s capture
Deep Space			
Earth $\rightarrow$ Kuiper Belt (40 AU)	7.5 km/s	30+ years	or Jupiter assist
Earth $\rightarrow$ solar escape	8.8 km/s	—	$v_\infty = 0$ at infinity
Earth $\rightarrow$ Voyager-like (17 km/s)	12.4 km/s	—	requires Jupiter assist realistically

Capture Δv assumes orbit insertion at 200 km altitude (or equivalent low orbit). Reduce with aerobraking where atmospheres exist.

6.7 ROTATION & ANGULAR MOMENTUM

6.7.1 Definitions

- **L (angular momentum):** Rotational analog of linear momentum. For a rotating body: $L = I\omega$. For a point mass: $L = mvr$ (perpendicular motion). Units: $\text{kg}\cdot\text{m}^2/\text{s}$. Conserved unless external torque applied.
- **I (moment of inertia):** Resistance to angular acceleration. Depends on mass distribution. Units: $\text{kg}\cdot\text{m}^2$.
- **ω (angular velocity):** Rotation rate. Units: rad/s . Convert from RPM: $\omega = \text{RPM} \times 2\pi/60$.
- **τ (torque):** Rotational force. $\tau = r \times F$ where r = lever arm, F = force. Units: $\text{N}\cdot\text{m}$. Causes angular acceleration: $\tau = I\alpha = dL/dt$.
- **α (angular acceleration):** Rate of change of angular velocity: $\alpha = \tau/I$. Units: rad/s^2 .

6.7.2 Moments of Inertia (Common Spacecraft Shapes)

All formulas assume uniform density. M = total mass, dimensions as labeled.

Shape	Formula	Example
Solid cylinder (spin axis)	$I = \frac{1}{2}MR^2$	Propellant tank, capsule
Hollow cylinder	$I = \frac{1}{2}M(R_{\text{out}}^2 + R_{\text{in}}^2)$	Stage shell
Solid sphere	$I = \frac{2}{5}MR^2$	Asteroid, fuel blob
Hollow sphere	$I = \frac{2}{3}MR^2$	Pressure vessel
Thin rod (center)	$I = \frac{1}{12}ML^2$	Boom, antenna
Thin rod (end)	$I = \frac{1}{3}ML^2$	Hinged panel
Rectangular plate (center)	$I = \frac{1}{12}M(a^2 + b^2)$	General case (rotation about center)

Shape	Formula	Example
Rectangular plate (edge)	$I = \frac{1}{3}Ma^2$ or $\frac{1}{3}Mb^2$	Solar panel tracking. <i>Use dimension perpendicular to rotation axis.</i>
Cube (center)	$I = \frac{1}{6}Ma^2$	Cubesat, box. Use parallel axis theorem for off-center axes.
Point mass at distance r	$I = Mr^2$	Reaction mass on tether

Parallel axis theorem: For axis offset by distance d from center of mass: $I_{\text{offset}} = I_{\text{cm}} + Md^2$.

Composite spacecraft: Sum moments of all components about spacecraft center of mass. Use parallel axis theorem for offset components.

6.7.3 Material Densities (Volume Sizing Reference)

For sizing containers, structures, and estimating mass from volume. Typical spacecraft average density: 500–1500 kg/m³ (mix of structure, electronics, propellant, empty space).

Material	Density (kg/m ³)	Notes
Water/Ice	1,000	Propellant, life support
Propellant (liquid)	800–1,000	Varies by type (hydrazine ~1,000, LOX ~1,140)
Aluminum	2,700–2,800	Lightweight structures (7075: 2,800)
Titanium	4,400–4,500	Aerospace structures (Ti-6Al-4V: 4,400)
Steel	7,800–7,900	Heavy structures (high-strength: 7,800)
Magnesium	1,740	Ultralight alloys
Rock/Silicates	2,000–3,000	Shielding, asteroid material
Carbon fiber composite	1,500–1,800	With matrix (T1000: 1,800)
Plastics/Organics	900–1,400	Varies by type
Kevlar	1,440	Aramid fiber
Typical spacecraft (average)	500–1,500	Mixed components, includes empty space

Sizing a 1-ton cube: For mass $M = 1000$ kg:

- Low density (500 kg/m³): side $a = 1.26$ m, volume = 2.0 m³
- Medium density (1000 kg/m³): side $a = 1.00$ m, volume = 1.0 m³
- High density (1500 kg/m³): side $a = 0.87$ m, volume = 0.67 m³

General formula: $a = \sqrt[3]{M/\rho}$ where ρ = average density.

6.7.4 Attitude Control

Thruster Sizing

Torque from thruster: $\tau = F \times r$, where F = thrust, r = distance from center of mass (lever arm).

Angular acceleration: $\alpha = \tau / I_{\text{spacecraft}}$

Time to rotate angle θ : Assuming bang-bang control (accelerate halfway, decelerate halfway):

$$t = 2\sqrt{\frac{\theta}{\alpha}} = 2\sqrt{\frac{\theta I}{Fr}}$$

Propellant per rotation: Each rotation requires two burns (start and stop). Total impulse = $2 \times F \times t_{\text{burn}}$ where $t_{\text{burn}} = \omega_{\text{max}}/\alpha$.

$$m_{\text{prop}} = \frac{2I\omega_{\text{max}}}{v_e \times r}$$

For small θ : $\omega_{\text{max}} = \sqrt{\alpha\theta} = \sqrt{Fr\theta/I}$

Example: 10,000 kg spacecraft, $I = 50,000 \text{ kg}\cdot\text{m}^2$, 100N thruster at 2m arm, 90° rotation. $\alpha = 200/50,000 = 0.004 \text{ rad/s}^2$. Time = $2\sqrt{1.57/0.004} = 40\text{s}$. $\omega_{\text{max}} = 0.08 \text{ rad/s}$. Propellant (hydrazine, $v_e = 2200 \text{ m/s}$) = $2 \times 50,000 \times 0.08 / (2200 \times 2) = 1.8 \text{ kg}$.

Reaction Wheels

Store angular momentum internally. No propellant consumed for routine maneuvers.

Wheel specs: Momentum capacity $L_{\text{max}} = I_{\text{wheel}}\omega_{\text{max}}$. Typical: 0.1–100 N·m·s per wheel.

Saturation: Wheels absorb angular momentum from disturbances (solar pressure, gravity gradient). Eventually saturate. Must “desaturate” using thrusters.

Momentum dumping: Thruster fires to transfer momentum from wheels to exhaust: $m_{\text{prop}} = L_{\text{accumulated}}/(v_e \times r)$.

CMGs (Control Moment Gyroscopes): Gimballed wheels. Output torque by changing orientation. Much higher torque than reaction wheels (100–1000× for same mass). Used on ISS, large spacecraft.

6.7.5 Artificial Gravity

Rotating Habitats

Centripetal acceleration: $a = \omega^2 r = v^2/r$

For 1g at radius r: $\omega = \sqrt{g/r}$, period $T = 2\pi\sqrt{r/g}$

Comfort limits: Human tolerance depends on rotation rate, not radius directly.

- **≤ 2 RPM:** Comfortable (requires radius $\geq 250\text{m}$ for 1g)
- **≤ 1 RPM:** Nearly imperceptible (radius $\geq 1000\text{m}$)
- **> 4 RPM:** Disorientation and nausea likely

Minimum radius for a target RPM: $r = 895/\text{RPM}^2$ meters (for 1g).

Coriolis effect: Apparent force on moving objects in rotating frame. Magnitude: $a_{\text{Coriolis}} = 2\omega v$. At 3 RPM moving 1 m/s: $a = 0.6 \text{ m/s}^2$ (noticeable, but tolerable).

Gradient: Gravity varies with radius. Head-to-foot difference at radius r, height h: $\Delta g/g = h/r$. At r = 100m, h = 2m: 2% difference.

6.8 TETHERS & ROTATING SYSTEMS

6.8.1 Tether Mechanics

Tension in a rotating tether: For a uniform tether of length L , mass M_{tether} , with tip mass M_{tip} , rotating at angular velocity ω :

$$T_{\max} = \omega^2 L \left(M_{\text{tip}} + \frac{M_{\text{tether}}}{2} \right)$$

Maximum tension at the hub. Tension varies linearly along length.

Characteristic velocity: Maximum tip speed a material can support under its own weight:

$$v_c = \sqrt{\frac{\sigma}{\rho}}$$

Where σ = tensile strength (Pa; 1 GPa = 10^9 Pa), ρ = density (kg/m^3). Result in m/s; divide by 1000 for km/s.

Tether mass ratio: Mass of tether needed per unit payload mass, for tip velocity v_{tip} :

v_{tip}/v_c	Mass ratio	Notes
0.3	0.2	Easy
0.5	0.6	Moderate
0.7	1.5	Practical limit
1.0	4.5	Heavy
1.5	30	Marginal
2.0	100+	Impractical

Practical limit: $\sim 0.7 \times v_c$ before tether mass becomes prohibitive. See Shop: Tether Materials for v_c values by material.

6.8.2 Skyhooks & Rotovators

A rotating tether in orbit can catch/release payloads at reduced relative velocity.

Tip velocity in orbit frame: If tether rotates so tip momentarily matches lower altitude, the tip velocity relative to ground is reduced.

Net Δv savings: Payload needs only reach tip velocity $v_{\text{orbit}} - v_{\text{tip}}$ instead of full orbital velocity.

Momentum exchange: Catching payload adds momentum to payload, subtracts from skyhook. The skyhook orbit decays. Must be reboosted (propellant, ion engines, or by catching mass thrown from orbit).

Simple rotovator example:

Circular orbit at altitude h , orbital velocity v_{orb} (from vis-viva). Tether length L (tip to center), tip velocity v_{tip} .

- Catch altitude: $h - L$
- Catch velocity (ground-relative): $v_{\text{orb}} - v_{\text{tip}}$
- Release altitude: $h + L$
- Release velocity: $v_{\text{orb}} + v_{\text{tip}}$

Example: 300 km orbit ($v = 7.73$ km/s), 200 km tether, tip at 1 km/s. Catch at 100 km altitude, payload needs 6.73 km/s (vs 7.8 km/s for direct orbit = 1+ km/s savings). Release at 500 km with 8.73 km/s—enough to reach Moon.

Tether rotation period: $T = 2\pi L/v_{\text{tip}}$. Must be synchronized with orbital period for repeated ground catches.

6.8.3 Stress & Safety

Factor of safety: Design strength = yield strength / safety factor. Typical: 2× for aerospace, 4× for crewed, 10× for life-critical.

Fatigue: Repeated loading weakens materials. For rotating systems: millions of cycles expected. Use fatigue strength (often 30–50% of tensile strength).

Micrometeorite damage: Tethers are vulnerable. Mitigation: redundant strands, sacrificial outer layers, larger cross-section. Flux varies with altitude; LEO is worst.

Thermal cycling: Space structures see $\pm 150^\circ\text{C}$ swings. Thermal stress = $E \times \alpha_T \times \Delta T$, where E = elastic modulus (Pa), α_T = thermal expansion coefficient (1/K). Check against yield strength.

7. Astronomical Data

7.1 REFERENCE CONSTANTS

- 1 AU = 149,600,000 km
- 1 light-year = 63,241 AU = 9.46 trillion km
- c (speed of light) = 299,792 km/s
- g_0 (standard gravity) = $9.81 \text{ m/s}^2 = 0.00981 \text{ km/s}^2$
- $G = 6.674 \times 10^{-20} \text{ km}^3/(\text{kg} \cdot \text{s}^2)$ – mass from μ : $M = \mu/G$

7.2 STAR DATA

Sun: $\mu = 133\text{B km}^3/\text{s}^2$, Distance = 0 AU (home star)

Proxima Centauri: $\mu = 16.3\text{B km}^3/\text{s}^2$ (0.123 solar masses), 4.24 light-years

Alpha Centauri A: $\mu = 145\text{B km}^3/\text{s}^2$ (1.10 solar masses), Distance = 4.37 light-years

Alpha Centauri B: $\mu = 119\text{B km}^3/\text{s}^2$ (0.90 solar masses), Distance = 4.37 light-years

Barnard's Star: $\mu = 22.3\text{B km}^3/\text{s}^2$ (0.168 solar masses), Distance = 6.0 light-years

Tau Ceti: $\mu = 104\text{B km}^3/\text{s}^2$ (0.783 solar masses), Distance = 11.9 light-years

Alpha Centauri A & B are binary pair orbiting each other; Proxima orbits them both

7.3 PLANETARY DATA

Mercury: Orbit = 0.39 AU, Radius = 2,440 km, $\mu = 22,032 \text{ km}^3/\text{s}^2$, Atmosphere = none, Period = 88 days, Position = 207° , Day = 58.6 days, Inclination = 7.0° , $e = 0.206$. 3:2 spin-orbit resonance (not 1:1). - *Composition:* 70% Iron-Nickel, 25% Silicates, 5% Sulfur. - *Temperature:* 100–700 K (extreme day/night variation).

Venus: Orbit = 0.72 AU, Radius = 6,052 km, $\mu = 324,859 \text{ km}^3/\text{s}^2$, Atmosphere = 16 km scale height / 65 kg/m^3 density, Period = 225 days, Position = 212° , Day = 243 days (retrograde), Inclination = 3.4° , $e = 0.007$. - *Composition:* 65% Silicates, 30% Iron-Nickel, 5% Trace metals. - *Atmospheric Composition:* 96.5% CO_2 , 3.5% N_2 , trace SO_2 , Ar, H_2O . - *Temperature:* 737 K (surface average, uniform due to thick atmosphere).

Earth: Orbit = 1.00 AU, Radius = 6,371 km, $\mu = 398,600 \text{ km}^3/\text{s}^2$, Atmosphere = 8.5 km scale height / 1.2 kg/m^3 density, Period = 365 days, Position = 42° , Day = 24.0 hours, Inclination = 0.0° , $e = 0.017$. - *Composition:* 65% Silicates, 32% Iron-Nickel, 2% Water, 1% Trace elements. - *Atmospheric Composition:* 78% N_2 , 21% O_2 , 0.9% Ar, 0.04% CO_2 , trace H_2O , Ne, He. - *Temperature:* 184–330 K (surface average 288 K).

- **Luna:** Orbit = 384,400 km, Period = 27.3 days, Radius = 1,737 km, $\mu = 4,903 \text{ km}^3/\text{s}^2$, Day = 27.3 days (tidally locked), Inclination = 5.15° , $e = 0.055$. Mascons cause significant orbital perturbations below ~100 km altitude.
 - *Composition:* 74% Silicates, 12% Iron-Nickel, 7% Aluminum, 4% Titanium, 2% Water Ice (polar craters), trace Helium-3.
 - *Temperature:* 40–396 K (extreme day/night variation).

Mars: Orbit = 1.52 AU, Radius = 3,390 km, $\mu = 42,828 \text{ km}^3/\text{s}^2$, Atmosphere = 11 km scale height / 0.02 kg/m³ density, Period = 687 days, Position = 223°, Day = 24.6 hours, Inclination = 1.85°, $e = 0.093$. Olympus Mons (22 km high), Valles Marineris (4000 km canyon system). - *Composition:* 70% Silicates, 25% Iron-Nickel, 3% Water Ice (at poles, subsurface permafrost), 2% Trace elements. - *Atmospheric Composition:* 95% CO₂, 2.7% N₂, 1.6% Ar, trace O₂, CO, H₂O. - *Temperature:* 130–300 K (surface average 210 K).

- *Phobos (D-type):* Orbit = 9,376 km, Period = 7.66 hours, Radius = 11 km, $\mu = 0.00071 \text{ km}^3/\text{s}^2$, Day = 7.66 hours (tidally locked), Inclination = 1.1°, $e = 0.015$. Orbits faster than Mars rotates (inside synchronous orbit). Very low Δv from Mars orbit.
– *Temperature:* ~233 K (average).
- *Deimos (D-type):* Orbit = 23,460 km, Period = 30.3 hours, Radius = 6 km, $\mu = 0.000098 \text{ km}^3/\text{s}^2$, Day = 30.3 hours (tidally locked), Inclination = 1.8°, $e \approx 0$.
– *Temperature:* ~233 K (average).

Jupiter: Orbit = 5.20 AU, Radius = 69,911 km, $\mu = 126,686,534 \text{ km}^3/\text{s}^2$, Atmosphere = 27 km scale height / 0.16 kg/m³ density, Period = 4333 days, Position = 90°, Day = 9.9 hours, Inclination = 1.3°, $e = 0.049$. - *Atmospheric Composition:* 89% H₂, 10% He, 0.3% CH₄, 0.3% H₂O, 0.2% NH₃, 0.2% Trace. - *Temperature:* 165 K (cloud top average). - *Radiation:* Intense radiation belts. See Environmental Hazards section.

- *Io:* Orbit = 421,700 km, Period = 1.77 days, Radius = 1,822 km, $\mu = 5,960 \text{ km}^3/\text{s}^2$, Day = 1.77 days (tidally locked), Inclination = 0.04°, $e = 0.004$. Thin SO₂ atmosphere ($\sim 10^{-7}$ bar). Extreme volcanism (surface constantly resurfaced, unstable for landing).
– *Composition:* 70% Silicates, 15% Sulfur, 12% Iron, 3% Sulfur Dioxide.
– *Temperature:* 90–130 K (surface average 110 K).
– *Radiation:* Very high (Jupiter's radiation belt).
- *Europa:* Orbit = 671,000 km, Period = 3.55 days, Radius = 1,561 km, $\mu = 3,203 \text{ km}^3/\text{s}^2$, Day = 3.55 days (tidally locked), Inclination = 0.47°, $e = 0.009$. Global subsurface ocean (15–25 km ice shell, 60–150 km deep). Chaos terrains indicate active geology.
– *Composition:* 55% Silicates, 25% Water Ice, 18% Iron-Nickel, 2% Salts.
– *Temperature:* 50–125 K (surface average 102 K).
– *Radiation:* High (Jupiter's radiation belt).
- *Ganymede:* Orbit = 1,070,400 km, Period = 7.15 days, Radius = 2,634 km, $\mu = 9,888 \text{ km}^3/\text{s}^2$, Day = 7.15 days (tidally locked), Inclination = 0.20°, $e = 0.001$. Has intrinsic magnetic field (reduces radiation exposure).
– *Composition:* 50% Water Ice, 35% Silicates, 13% Iron-Nickel, 2% Organics.
– *Temperature:* 70–152 K (surface average 110 K).
– *Radiation:* High (Jupiter's radiation belt, partially shielded by own field).
- *Callisto:* Orbit = 1,882,700 km, Period = 16.7 days, Radius = 2,410 km, $\mu = 7,179 \text{ km}^3/\text{s}^2$, Day = 16.7 days (tidally locked), Inclination = 0.28°, $e = 0.007$.
– *Composition:* 55% Water Ice, 32% Silicates, 10% Iron-Nickel, 3% Organics.
– *Temperature:* 80–165 K (surface average 134 K).
– *Radiation:* Low (outside main radiation belt).

Saturn: Orbit = 9.54 AU, Radius = 58,232 km, $\mu = 37,931,187 \text{ km}^3/\text{s}^2$, Atmosphere = 60 km scale height / 0.19 kg/m³ density, Period = 10759 days, Position = 331°, Day = 10.7 hours, Inclination = 2.5°, $e = 0.057$. Has rings (avoid close equatorial approaches). - *Atmospheric Composition:* 96% H₂, 3% He, 0.4% CH₄, 0.2% H₂O, 0.2% NH₃, 0.2% Trace. - *Temperature:* 134 K (cloud top average).

- *Titan:* Orbit = 1,221,870 km, Period = 16.0 days, Radius = 2,575 km, $\mu = 8,978 \text{ km}^3/\text{s}^2$, Atmosphere = 21 km / 5.3 kg/m³ (1.5× Earth pressure), Day = 16.0 days (tidally locked), Inclination = 0.35°, $e = 0.029$. Liquid methane/ethane lakes and seas on surface (affects landing site selection).
– *Composition:* 45% Silicates, 40% Water Ice, 6% Methane/Ethane, 5% Organics, 4% Ammonia.
– *Atmospheric Composition:* 98% N₂, 1.4% CH₄, trace H₂, Ar, C₂H₆, C₂H₂.
– *Temperature:* 94 K (surface average).

- **Enceladus:** Orbit = 237,950 km, Period = 1.37 days, Radius = 252 km, $\mu = 7.2 \text{ km}^3/\text{s}^2$, Day = 1.37 days (tidally locked), Inclination = 0.01° , $e = 0.005$. Subsurface ocean accessible via south polar geysers. Geysers create Saturn's E ring (orbital debris hazard).
 - *Composition:* 70% Water Ice, 22% Silicates, 3% Ammonia, 3% Organics, 2% Salts.
 - *Temperature:* 32–145 K (surface average 75 K).

Uranus: Orbit = 19.2 AU, Radius = 25,362 km, $\mu = 5,793,939 \text{ km}^3/\text{s}^2$, Atmosphere = 28 km scale height / 0.42 kg/m^3 density, Period = 30689 days, Position = 329° , Day = 17.2 hours, Inclination = 0.77° , $e = 0.046$. Axial tilt = 98° (extreme seasons, affects solar power). - *Atmospheric Composition:* 83% H_2 , 15% He, 2% CH_4 . - *Temperature:* 76 K (cloud top average).

Neptune: Orbit = 30.06 AU, Radius = 24,622 km, $\mu = 6,836,529 \text{ km}^3/\text{s}^2$, Atmosphere = 20 km scale height / 0.45 kg/m^3 density, Period = 60182 days, Position = 320° , Day = 16.1 hours, Inclination = 1.77° , $e = 0.009$. - *Atmospheric Composition:* 80% H_2 , 19% He, 1% CH_4 . - *Temperature:* 72 K (cloud top average).

- **Triton:** Orbit = 354,760 km, Period = 5.88 days (retrograde), Radius = 1,353 km, $\mu = 1,428 \text{ km}^3/\text{s}^2$, Day = 5.88 days (tidally locked), Inclination = 157° (retrograde), $e \approx 0.00001$. Active nitrogen geysers (affects landing operations).
 - *Composition:* 50% Silicates, 35% Water Ice, 5% Nitrogen Ice, 5% Methane Ice, 5% CO_2 Ice.
 - *Temperature:* 38 K (surface average).

Position = heliocentric longitude (degrees from vernal equinox). Positions as of 2025-11-28. Use Physics Handbook: Launch Windows to calculate phase angles. Atmosphere density = surface (or cloud deck for gas giants). Scale height H = altitude where density drops to 37%.

7.4 EARTH ORBITAL REGIMES

F = failures per 1000 days (kd). Add to equipment F . See *Chance, Shop: Markets, Environmental Hazards*.

7.4.1 Orbital Bands & Launch Traffic

Band	Altitude	Inclination	Δv from 200 km	Mass/yr	Rideshare
LEO (low)	200–400 km	51° , varied	0	~1,500–2,500t	Yes — 50–831 kg/port. See Shop: Rideshare
LEO (mid/SSO)	400–1,200 km	98° (SSO), mixed	+0.1–0.5 km/s	—	Transporter ~4×/yr; ~200–400t addressable
MEO	~20,200 km	$55^\circ, 63^\circ$	+2.4 km/s	~10–20t	Rare
GEO	35,786 km	$\sim 0^\circ$	+3.9 km/s	~80–150t	Rare
L2	1.5M km	—	+3.2 km/s	~50–100t beyond	Occasional

~250 launches/yr, ~2,800 sats. Debris F : LEO congested $2/10\text{m}^2$, sparse $0.5/10\text{m}^2$, MEO/GEO $0.1/10\text{m}^2$.

7.4.2 High-Value Asset Classes

Class	Cost	Fleet	Mass	Altitude	Incl	F/kd	Servicing
Crewed stations	\$100–150B sys	1–2	~420t	~400 km	51°	0.001–0.01	EVA only
Commercial GEO	\$300–500M	~500	5–7t	GEO	0°	0.02–0.06	MEV ~80%
OPIR	\$3–5B	8–12	4–6t	GEO/HEO	0°/63°	0.01–0.04	Classified
Optical ISR	\$0.6–4B	10–15	15–20t	250–1,000 km	98°	0.02–0.08	NRO
Radar ISR	\$1–3B	8–15	2–16t	500–900 km	57°–68°	0.03–0.10	SAR
GNSS	\$250–550M	90–120	2–4t	20,200 km	55°	0.003–0.02	MEO
LEO broadband	\$0.25–2M	10k–40k	250–800 kg	340–1,200 km	mixed	0.05–0.20	Replace, not repair
Protected mil GEO	\$0.5–2B	10–15	~6t	GEO	0°	0.01–0.04	MEV
Weather	\$0.8–1.5B	15–25	2–5t	LEO/GEO	SSO/0°	0.01–0.05	—
Flagship scopes	\$5–10B	<10	6–12t	L2	—	0.005–0.03	None
On-orbit servicing	\$300–700M	10–20	1–3t	LEO/GEO	varied	0.03–0.15	MEV/MRV
Deep-space relays	\$1–3B	5–8	1–3t	non-Earth	—	0.01–0.06	Piggyback

7.4.3 Demand (Shop: $D = N\ddot{O}(A/P)^E$)

Capability	N/yr	A	E
GEO comms	15–25	\$300–500M	0.5–1
LEO broadband	2k–4k	\$0.5–1M	1–2
EO/remote sensing	50–100	\$50–200M	1
GNSS	2–4	\$300M	0
Weather	1–3	\$1B	0
On-orbit servicing	2–5	\$50–150M	1–2
Rideshare	500–1k sats	\$6–10k/kg	1.5

7.4.4 Repair Notes

GEO: ~80% MEV-compatible; life extension \$50–150M for 5–15 yr. **LEO:** Stations = EVA; broadband = replace. **MEO/L2:** No servicing. **Economics:** Replacement cost = bid ceiling. See Shop: Contracts.

7.5 ASTEROID BELT

Located between Mars and Jupiter, roughly 2.2 to 3.3 AU from Sun. Volume $\approx 20 \text{ AU}^3$. Density $\approx 50,000$ asteroids (>1 km) per AU^3 .

C-type (75%): $\rho = 1.3 \text{ g/cm}^3$. 50% Silicates, 22% Water Ice, 15% Clays, 10% Organics, 3% Metals.

S-type (17%): $\rho = 2.7 \text{ g/cm}^3$. 76% Silicates, 17% Iron-Nickel, 5% Magnesium, 2% Trace.

M-type (8%): $\rho = 5.3 \text{ g/cm}^3$. 82% Iron, 10% Nickel, 6% Silicates, 2% Cobalt, 0.05% PGMs. PGMs = Platinum Group Metals.

D-type (outer belt, Trojans, Phobos/Deimos): $\rho = 1.5 \text{ g/cm}^3$. 48% Silicates, 20% Organics, 20% Water Ice, 10% Carbon, 2% Metals.

Specific composition of any undiscovered asteroid is unknown until surveyed. Use Chance mechanics: propose expected composition, negotiate with other player, roll to determine actual percentages.

Ceres (C-type): Orbit = 2.77 AU, Radius = 476 km, $\mu = 63 \text{ km}^3/\text{s}^2$, Period = 1,680 days, Position = 115°, Day = 9.1 hours, Inclination = 10.6°, $e = 0.076$. Bright spots (salt deposits) in Occator crater. Possible subsurface ocean. - *Temperature: ~167 K (average).*

Vesta (S-type): Orbit = 2.36 AU, Radius = 263 km, $\mu = 17 \text{ km}^3/\text{s}^2$, Period = 1,325 days, Position = 298°, Day = 5.3 hours, Inclination = 7.1°, $e = 0.089$. Differentiated body (core, mantle, crust) — unusual for asteroid. - *Temperature: ~181 K (average).*

Pallas (C-type): Orbit = 2.77 AU, Radius = 256 km, $\mu = 13 \text{ km}^3/\text{s}^2$, Period = 1,686 days, Position = 22°, Day = 7.8 hours, Inclination = 34.8°, $e = 0.231$. - *Temperature: ~167 K (average).*

Hygiea (C-type): Orbit = 3.14 AU, Radius = 217 km, $\mu = 5.8 \text{ km}^3/\text{s}^2$, Period = 2,029 days, Position = 184°, Day = 13.8 hours, Inclination = 3.8°, $e = 0.117$. - *Temperature: ~155 K (average).*

These four comprise over half the belt's total mass.

7.6 KUIPER BELT

Located beyond Neptune, roughly 30 to 50 AU from Sun. Volume $\approx 2000 \text{ AU}^3$. Density ≈ 50 objects (>100 km) per AU^3 . Much sparser than main belt.

KBO-type: $\rho = 1.8 \text{ g/cm}^3$. 45% Water Ice, 43% Rock/Silicates, 5% Methane Ice, 4% Organics, 2% Ammonia Ice, 1% Nitrogen Ice.

Pluto: Orbit = 39.48 AU, Radius = 1,188 km, $\mu = 870 \text{ km}^3/\text{s}^2$, Atmosphere = trace N_2 (variable), Period = 90560 days, Position = 2°, Day = 6.39 days (tidally locked with Charon), Inclination = 17.2°, $e = 0.249$. Sputnik Planitia (large nitrogen ice plain, affects landing). - *Composition:* 55% Rock/Silicates, 35% Water Ice, 5% Organics, 3% Nitrogen Ice, 2% Methane Ice. - *Atmospheric Composition:* 98% N_2 , 1.5% CH_4 , 0.5% CO (variable with distance from Sun). - *Temperature:* 33–55 K (surface average 44 K, varies with distance).

- **Charon:** Orbit = 19,591 km, Period = 6.39 days, Radius = 606 km, $\mu = 102 \text{ km}^3/\text{s}^2$, Day = 6.39 days (tidally locked), Inclination $\approx 0^\circ$, $e \approx 0$.
 - *Composition:* 55% Water Ice, 40% Rock/Silicates, 3% Ammonia Ice, 2% Organics.
 - *Temperature:* ~53 K (average).

Eris: Orbit = 67.8 AU, Radius = 1,163 km, $\mu = 1,108 \text{ km}^3/\text{s}^2$, Atmosphere = none, Period = 203,830 days, Position = 24°, Day = 25.9 hours, Inclination = 44.2°, $e = 0.441$. - *Composition:* 60% Rock/Silicates, 30% Water Ice, 4% Methane Ice, 4% Organics, 2% Nitrogen Ice. - *Temperature:* ~30 K (average).

- **Dysnomia:** Orbit = 37,300 km, Period = 15.8 days, Radius = 350 km, $\mu \approx 5 \text{ km}^3/\text{s}^2$, Day = 15.8 days (tidally locked), Inclination $\approx 78^\circ$, $e \approx 0.006$. Composition similar to Eris.
 - *Temperature:* ~30 K (average).

Makemake: Orbit = 45.4 AU, Radius = 715 km, $\mu = 200 \text{ km}^3/\text{s}^2$, Atmosphere = none, Period = 111,845 days, Position = 97°, Day = 22.8 hours, Inclination = 29.0°, $e = 0.159$. - *Composition:* 55% Rock/Silicates, 25% Water Ice, 8% Methane Ice, 7% Organics, 5% Ethane Ice. - *Temperature:* ~32 K (average).

Haumea: Orbit = 43.1 AU, Radius = 816 km (elongated), $\mu = 267 \text{ km}^3/\text{s}^2$, Atmosphere = none, Period = 103,410 days, Position = 212°, Day = 3.9 hours, Inclination = 28.2°, $e = 0.195$. Has rings. - *Composition:* 80% Water Ice, 18% Rock/Silicates, 2% Organics. - *Temperature:* ~32 K (average).

7.7 ENVIRONMENTAL HAZARDS

Add F values based on location and duration. Apply shielding reductions. Sum with equipment F. See Chance for total risk.

7.7.1 Solar cycle (from in-world year)

The Sun's activity varies on a **~11-year cycle** (real cycles are often **9–14 years**). High activity **heats and expands Earth's thermosphere**, so **LEO atmospheric drag** is much higher at **solar maximum** than at **solar minimum** — see Physics Handbook: **Station-Keeping**. The same cycle drives **more solar particle events** outside the magnetosphere (**Interplanetary** radiation, below).

References (optional calibration): [WDC-SILSO sunspot numbers](#), [NOAA solar cycle progression](#). Use your campaign **calendar year** Y (or map fiction to an equivalent CE year).

Cycle index τ — remainder of $(Y - 2020)$ modulo 11, in **0 ... 10** (if negative, add 11 until it falls in that range). *2020 is anchored near **Solar Cycle 25** minimum; shift the anchor year if you want another epoch to line up with SILSO/NOAA.*

τ	Phase	Interplanetary radiation	LEO drag makeup Δv (same β as handbook)
0, 1, 10	min	<i>solar min</i> row (below)	Solar min column
4, 5, 6	max	<i>solar max</i> row (below)	Solar max column
2, 3, 7, 8, 9	moderate	0.4 Crew F/kd, 0.4 Elec F/kd	$\Delta v \approx \frac{1}{2}(\Delta v_{\min} + \Delta v_{\max})$ for that altitude

Between phases (optional): treat activity as a weight $s \in [0, 1]$ from min $\rightarrow$ max (e.g. $s = 0$ at **min**, $s = 1$ at **max**, $s = \frac{1}{2}$ at **moderate**). Then **LEO drag makeup** $\Delta v \approx (1 - s) \Delta v_{\min} + s \Delta v_{\max}$ using the handbook row for your altitude. Interplanetary F/kd can scale similarly between **0.2** and **0.6** (e.g. $0.2 + 0.4s$ for Crew and Elec).

Multi-year missions: use the phase for the year that dominates the exposure, or **max** of years transited for a conservative budget.

*Van Allen / Jupiter / LEO trapped belts: unchanged by solar cycle in this ruleset. **Interplanetary** radiation and **LEO drag** are the phase-dependent knobs.*

7.7.2 Radiation

Zone	Crew F/kd	Elec F/kd
LEO	0.01	0.1
Van Allen transit	0.5/pass	2.0/pass
Interplanetary (solar min)	0.2	0.2
Interplanetary (solar max)	0.6	0.6
Jupiter	10.0	10.0

Interplanetary: Use **solar min** or **solar max** row according to **Solar cycle** above; **moderate** phase uses **0.4 / 0.4** (not duplicated as a table row).

Protection: Crew shielding (0.5t/crew, $\times 0.5$), Hardening ($2\times$ cost, $\times 0.3$), Electronics shielding (0.5t/m³, $\times 0.5$). See Shop. See Odds: Radiation Upset, Radiation Exposure.

7.7.3 Debris & Micrometeorites

Zone	F/10m ²
LEO (congested)	2.0
LEO (sparse)	0.5
MEO/GEO	0.1
Interplanetary	0.05
Asteroid belt	0.2
Outer system	0.01

Protection: Whipple (20kg/m², ×0.3). See Shop. See Odds: Micrometeorite Damage, Pressure Leak.

7.7.4 Thermal

Condition	F/kd
Cycling	$0.3 \times (\text{cycles/day} \times \text{days} / 1000)$
Extreme cold (<50K)	0.5
Extreme heat (>400K)	1.0

Cycles: LEO 16/day, tumbling 1-2/day, stable attitude 0. **Protection:** MLI (1kg/m², ×0.5), active cooling, attitude control. See Shop.

7.7.5 Atmospheric

Environment	F
Earth entry	50
Mars entry	30
Venus/Titan	80
Aerobraking	20/pass

Requires: Heat shield (15% entry mass). See Shop. See Odds: Heat Shield Failure.

7.7.6 Example

Mars mission (500 days, 2 crew, 40m², crew shield + hardened avionics + Whipple): - Crew radiation: $0.2/\text{kd} \times 0.5\text{kd} \times 0.5 = 0.05$ - Electronics: $0.2/\text{kd} \times 0.5\text{kd} \times 0.3 = 0.03$ - Micrometeorites: $0.05/10\text{m}^2 \times 4 \times 0.5\text{kd} \times 0.3 = 0.03$ - **Total environmental F: 0.11** (add equipment F + one-shot F)

8. Chance

*Propose what might exist. Argue for the odds. Roll to learn **when** it happens — or **that** it doesn't.*

Shop **D**; **P** lines need a breakthrough first (D in USD, P = chance it's possible). Reliability uses Shop fragility F ; search uses Astrodata density δ . Research: $F = 1/D$ with effort in the same USD unit as D .

8.1 One model

1. **Maybe it's real.** Negotiated $P \in [0, 1]$: the phenomenon *exists* this campaign. If not, no effort makes it happen. Known hardware and accepted physics: $P = 1$.
2. **If it's real, effort pulls toward an outcome.** F is event rate per unit effort — larger $F \Rightarrow$ things happen sooner or more often per unit spent.

Effort x is domain-specific (kilodays, lab dollars, AU^3 , ...). The dimensionless product is always Fx (matched units).

8.2 Rolling *when* (or *if*)

Roll d100 (00–99) to get a uniform $u \in [0, 1)$. Compute:

$$X = \frac{-\ln(1 - u/P)}{F}$$

- u — from the d100: start with roll/100 (so $45 \rightarrow 0.45$). A d100 only names a **1% bin**. If you rolled **00** or **99** and the next digits can change the outcome, roll again for the next two digits; repeat on another edge. 00 *then* 12 $\rightarrow u = 0.0012$. 99 *then* 37 $\rightarrow u = 0.9937$ (*opens the long tail past $X \approx 4.6$*).
- P — chance it *can* happen ($P = 1$ for ordinary failures and catalog gear).
- F — event rate per unit effort (same unit as commitment and X).
- X — effort that would have produced the outcome (time to failure, dollars to breakthrough, volume to detection, ...).

Resolve

- $u \geq P \rightarrow$ impossible branch; **does not happen**.
- Else if commitment $\geq X \rightarrow$ **happens** when effort reaches X .
- Else $\rightarrow$ **not yet** this commitment.

Engine ($F = 2.5/\text{kd}$, $P = 1$), 1 kd mission, roll 45: $X = -\ln(0.55)/2.5 \approx 0.24 \text{ kd} \rightarrow$ failure at day 240.

Search ($P = 0.5$, $F = 0.1/\text{AU}^3$), 10 AU^3 , roll 30: $X = 9.2 \text{ AU}^3 \rightarrow$ found. Roll 55 (≥ 50) $\rightarrow$ impossible. Roll 45: $X = 23 \text{ AU}^3 \rightarrow$ not in this search.

Planning (no dice): $\Pr(\text{by effort } x) = P(1 - e^{-Fx})$. At $x = 1/F$, about 63%; at $3/F$, about 95% (when $P = 1$).

8.3 Three domains

Domain	Effort x	P	F
Reliability	kd (operating)	1	Shop fragility
Research	USD (same unit as D)	negotiate	$F = 1/D$
Search	AU ³	negotiate	Astrodata δ

Search volume $\approx \pi \times \text{sensor_range}^2 \times \text{path_length}$ (Shop: Sensors).

Speculative: propose $\rightarrow$ argue $\rightarrow$ other player sets P and F (or D for research).

8.4 Systems

Serial (any failure fails the stack): rates add. $F_{\text{total}} = F_1 + F_2 + \dots$

Engine 2.5 + avionics 0.5 + seals 5 $\rightarrow$ 8/kd.

Redundancy (N identical units, need K working): each unit keeps Shop F . Stack fails only after $m = N - K + 1$ unit failures. Do **not** collapse the bank into one rate for $1 - e^{-Et}$ — that matches mean lifetime, not short-burn hazard, and can make engine-out worse than single-string.

- **At the table:** while n work, bank rate $= nF$ (next unit failure). On failure: $n \leftarrow n - 1$; if $n < K$, stack lost; else continue. Odds (F) for the unit that failed.
- **Planning:** each unit fails with $p = 1 - e^{-Ft}$; stack fails if $\geq m$ fail (binomial). If $Ft \ll 1$, $P(\text{loss}) \approx \binom{N}{m} p^m$.

Nine engines, $F = 120/\text{kd}$, 180 s ($t \approx 2.1 \times 10^{-6}$ kd), need 7 ($m = 3$): $p \approx 2.5 \times 10^{-4}$, $P(\geq 3) \sim 10^{-9}$.

Long-life aside (not for mission E): mean time to cluster loss $= (1/F) \sum_{i=K}^N 1/i$.

One-shots ($\dagger$): Shop % as decimal p (e.g. 0.25% $\rightarrow p = 0.0025$). Same u as above; fail if $u < p$. 5%: *fail on 00–04*. 0.25%: *first roll ≥ 01 pass; 00 then fail if second < 25* . Odds ($\dagger$) — not the continuous (F) table. Planning: add each p ; redundant $\dagger$: binomial, not summed.

8.5 Mission simulation

One-shots — At each activation, roll per $\dagger$; on failure $\rightarrow$ Odds ($\dagger$); resolve before continuing.

Continuous

1. **Manifest** — split the mission into **segments** (phases) in which the same parts are in use. List in-use parts with F . Idle = 0. Redundant bank: one line at rate nF .
2. **Exposure** — segment rate $F_{\text{seg}} = \sum F$; $E = F_{\text{seg}} \times t$. A full bank contributes nF (next unit failure, not stack loss).
3. **Roll** — d100 $\rightarrow u$ (edge-refine 00/99 if it matters); $X = -\ln(1 - u)$. If $X > E$, no continuous failure this segment.
4. **When / which** — failure time $= X/F_{\text{seg}}$ into the segment. Which line: roll d100 against each line's share F/F_{seg} .
5. **How** — Odds (F) (never ignition/deploy modes).
6. **Handle** — abort / backup / degraded. Bank still $\geq K$ working $\rightarrow$ not stack loss; reduce n .
7. **Repeat** from failure time with survivors.

Planning: single-string $\approx \sum (Ft) + \sum \dagger$. Banks: binomial under Redundancy, not nFt as stack-loss probability.

Mars lander, 0.003 kd. Two engines need 1 ($F = 10/\text{kd}$ each) $\rightarrow$ bank 20/kd, $E = 0.060 + \text{avionics } 0.0003$. Roll 08 $\rightarrow X = 0.083 > 0.0603 \rightarrow$ no failure. A first engine-out leaves one unit; stack loss only on a second.

8.6 Many events

$E = F \times \text{effort}$ (expected *count*).

- $E < 5$: sample gaps with the core formula; accumulate until past commitment.
- $E \geq 5$: $\text{Count} \approx E \times (\text{avg of 2 d100})/50$.

$F = 0.05/\text{AU}^3$, $1000 \text{ AU}^3 \rightarrow E = 50$; *rolls* 34, 62 $\rightarrow$ *count* ≈ 48 .

Annual rates: set effort to one year of the relevant unit, then use the same split.

8.7 One draw vs. coming back ($P < 1$)

Default: one d100 per research/discovery line — fixes impossible or X . Track cumulative effort; never re-roll existence each quarter.

Optional re-roll after spend x without success: update first,

$$P_{\text{new}} = \frac{e^{-Fx} P_{\text{old}}}{e^{-Fx} P_{\text{old}} + (1 - P_{\text{old}})}$$

then roll again. Skip the update and you stack independent P draws.

8.8 Appendix: other dice & generation

Type	Use
Uniform	d100 vs threshold
Long tail	Roll until $\leq$ threshold, count
Log-uniform	$10^{\text{roll}/25}$ (order-of-magnitude)
Clustered	Average of 2 rolls

$P(A \cap B) = P(A) P(B \mid A)$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $P(\neg A) = 1 - P(A)$.

Tables: start anywhere; two digits $\rightarrow$ up (tens), right (ones), wrap. Next cell = next roll.

Seed: $(\text{seed} \times 21 + 1) \bmod 10000$; first two digits = roll; all four = next seed.

Dice: d100 or two d10 (tens, then ones).

Guess: for 1% one-shots, hide a d100; player picks 0–99; match = fail.

9. Odds

When something fails, look it up here. Italicized names are cross-references to other entries.

Continuous (F) vs activation (†) — Do not mix these.

Rolled from	Use entries marked	Never
F (burn-days / operating effort)	(F) — failures while the system is already in use	Ignition, deploy, latch, first-light, cold-start
† (Shop % at activation)	(†) — failures at light, relight, deploy, separate, entry, landing	Wear-out / mid-cruise erosion as if it were a start fault

Shop lines with both † and F point at **two** Odds entries. Continuous F only accrues while the part is doing relevant work (thrusting, tracking powered, radiator rejecting heat, etc.) — an unused panel arm does not roll *Actuator failure*.

F MODIFIERS — Multiply Shop F values before calculating mission risk.

Heritage	Units Built	F Multiplier	Cost Multiplier
Prototype	1–5	×10	×5
Early	6–20	×3	×3.5
Established	21–100	×1	×2.5
Mature	101+	×0.3	×1

Hardening: ×0.3 F (2× cost). Rad-hard + redundant internals. Declare at purchase.

Actuator failure (F) — Motor burnout (mechanism frozen) 40%, Gear wear (slow/weak operation) 30%, Sensor feedback loss (uncontrolled motion) 20%, Wiring fault → *Electrical fire* 10%. *Only while the actuator is in use (e.g. panel tracking while driving). Idle arm → no F exposure. Crew: manual operation if designed for it. Often repairable with spares (2hr).*

Airlock failure — *Seal failure* 50%, *Actuator failure* (hatch jam) 30%, Pressure sensor fault (false readings) 15%, Explosive decompression (vehicle damage, crew loss if occupied) 5%. *Crew: manual seal + crank if jammed. Never cycle with faulty sensors.*

Antenna failure — Pointing error (data rate -50%) 60%, Mechanical jam (must reorient entire spacecraft to communicate) 40%. *Crew: realignment EVA (2hr) fixes. Ground: DSN boost can work around pointing error.*

Attitude control failure — *Reaction wheel failure* 40%, *CMG failure* 25%, *RCS failure* 25%, *Software fault* 10%. *Spacecraft may tumble if all control lost. Solar panels lose sun-pointing; thermal limits exceeded in hours.*

Atmospheric control failure — *CO₂ scrubber failure* 50%, *O₂ generator failure* 30%, *Pressure leak* 20%

Avionics failure — *Flight computer failure* 33%, *Communications failure* 33%, *Guidance/navigation failure* 25%, *Software fault* 9%

Battery failure — Cell degradation (capacity -30%) 50%, Thermal runaway → *Electrical fire* 25%, Controller fault (won't charge/discharge) 20%, Short circuit (immediate offline) 5%. *Isolate failed pack. Reduce loads to match remaining capacity.*

Burn-through (†) — Thermal protection breached. Vehicle loss during reentry.

Casing breach — Case, field joint, or insulation/bond burns through; motor vents sideways or ruptures. Vehicle loss (or that stage destroyed). *Solid motors cannot be inspected in flight. Quality control is the only defense. Multi-body stacks: adjacent stages/payload may be damaged — negotiate survivors only if fiction clearly separates them.*

Camera failure — Image degradation (resolution -50%, noise increase) 50%, Sensor noise (hot pixels, artifacts) 25%, Pointing mechanism jam (fixed orientation) 15%, Complete failure (no images) 10%. *Degraded/noisy: continue with reduced quality. Jam: reorient spacecraft to image target. Complete: switch to backup camera or continue mission blind.*

Catalyst bed degradation — Hydrazine catalyst granules sinter, void, or attrit after long pulse/SS life. Thrust -20%, Isp -10%; roughness / chamber-pressure noise. *Thruster still usable. Switch to backup if timing-critical. No in-flight repair (catalyst replace on ground).*

Cell degradation — Power output -10% immediately, then -5% per year thereafter (automatic, no roll). *Reduce power draw. No repair possible.*

Chamber degradation — Ablative or regeneratively cooled chamber wall wears / heats unevenly. Isp -10%. *Engine still usable — recalculate Δv / burn time. Not repairable in flight.*

Channel erosion — Isp -10%, thrust -8% immediately, then -3% Isp per year thereafter (automatic, no roll). *Hall wear-out: channel wall, insulator, or magnetic-pole cover thinned by plasma. Thruster still usable; recalculate transfer times.*

Chemical engine failure (F) — *Combustion instability 35%, Turbopump failure 35%, Nozzle failure 22%, Injector failure 8%. Mid-burn only — engine already lit. Not repairable in flight. Start/relight → Chemical ignition failure. Rutherford (electric pump): on Combustion, 50% → treat as pump/power stop (engine offline) instead. Vulcain-class GG hydrolox: if fiction fits, prefer Nozzle over Combustion.*

Chemical ignition failure (†) — *Ignition failure 50%, Valve stuck 30%, Pump/power fail to spool (engine offline this burn; treat as turbopump seizure) 20%. Shop † at light or relight only. Never from burn-day F.*

CMG failure — Bearing wear (torque output -50%) 40%, Motor failure (offline) 30%, Gimbal jam (fixed orientation) 20%, Software fault 10%. *Switch to backup CMG or RCS. Crew: bearing replacement possible (4hr) with spares.*

CO₂ scrubber failure — Saturation: 8hr to crew incapacitation, 12hr to death (double if crew rests). *Crew: swap cartridge (1hr). Spare cartridges: 5kg, \$1k each; plan 1 per 30 crew-days.*

Combustion instability — Chamber pressure oscillates (injector/acoustics / mixture-ratio wander). *Oscillation 60%, Shutdown required (engine offline for mission) 30%, Destructive → Propellant feed failure 10%. Throttle to 50% may stabilize Oscillation. Shutdown: switch to backup engine or abort burn. Destructive: feed/pump hardware damaged — follow that table.*

Communications failure — *Antenna failure 40%, Transmitter failure 30%, Receiver failure 20%, Total loss (no comms; spacecraft follows pre-programmed sequence only) 10%*

Coronagraph failure — Stray light contamination (image quality -50%, can still detect bright CMEs) 35%, Occulter misalignment (solar disk bleeds through, data unusable until realigned) 25%, *Instrument failure 25%, Door/cover mechanism jam (instrument blind) 15%. Misalignment: ground command realignment sequence (4hr, 60% success). Door jam: crew EVA if accessible, otherwise instrument lost.*

Cooling failure — Regen / film cooling can't reject chamber heat (blocked channel, burned tube, pump-fed coolant loss). *Shutdown required or the chamber melts. Engine offline for remainder of mission. Solids never roll this — see Nozzle failure.*

Cooling loop failure — Hot zones develop. *Reduce power draw 30%, or Avionics failure occurs automatically in 6hr. Crew: reroute coolant if backup loops. Rotate spacecraft for passive thermal control.*

Cryocooler failure — Reduced capacity (cooling output -50%, boiloff doubles) 50%, Compressor bearing wear (capacity -30%, vibration, accelerates wear) 25%, Cold head contamination (offline) 15%, Complete failure (revert to passive boiloff) 10%. *Reduced capacity: continue with higher boiloff. Bearing wear: continues operation but F doubles going forward. Contamination: crew purge cycle (8hr, 60% success) or requires replacement. Complete: revert to passive*

boiloff rates per propellant type.

Deployment jam (chute) (†) — Chute offline; use backup. *Landing activation. If no backup: retro-rockets required, or 80% chance of Structure failure* on landing.**

Deployment jam (gear) (†) — Gear stuck. *Landing activation. Crew: manual crank (1hr, if designed) or EVA unjam (3hr). If uncleared: belly landing, 60% chance of Structure failure.*

Deployment jam (panels) (†) — Partial deployment (power -50%) 80%, Complete jam (power from RTG/batteries only) 20%. *First deploy only. Crew: EVA to clear (2hr). May require cutting debris or manual crank.*

Docking hardware failure — Electromagnet misalignment (capture miss, re-approach) 40%, Field strength degradation (grip weak, capture mass limit halved) 25%, Power fault during approach (abort, retry) 20%, Control electronics fault (offline) 15%. *Miss: re-approach costs time and RCS propellant. Degradation: can still capture lighter objects. Electronics: reboot (1hr, 70% success) or replace controller with spares (4hr EVA).*

Drone failure — Battery failure 40%, Motor failure (uncontrolled) 30%, Communications failure (lost contact) 20%, Collision (destroyed) 10%. *Drones are expendable. Recover if possible; replace if not.*

Electrical fire — Suppressed automatically (no further damage) 60%, Spreads → roll Avionics failure 30%, Major (power -50% permanently) 10%. *Crew: isolate circuits to prevent spread.*

Electrolysis failure — Filter clog (offline, 1 day to replace filter) 35%, Hydrogen sensor fault (unsafe operation, must replace) 25%, Pump failure (offline, 2 days to replace) 20%, Cell stack degradation (output -40%, continue or replace) 15%, Controller fault (reboot or replace) 5%. *Repairs require spares (5-15% unit cost). Multiple failures common over system lifetime.*

Electronic failure (generic) — Component failure (offline, switch to backup) 50%, Software fault 25%, Connector/wiring fault (intermittent) 15%, Radiation upset 10%. *Most electronics have redundancy. Single-string systems: negotiate specific consequence.*

EVA suit failure — Thermal regulation failure 35%, Pressure leak (suit) 30%, Joint failure (mobility impaired) 20%, Visor damage (abort EVA) 10%, Life support failure (15 min emergency reserve) 5%. *Return to airlock immediately on any suit failure. Buddy system essential.*

Factory failure — Mechanical failure 40%, Thermal failure (process temp) 25%, Feedstock jam 20%, Control system fault 10%, Catastrophic (containment breach, facility damaged) 5%. *Offline for repair: 1 day per \$10M facility value. Catastrophic may injure crew.*

Fatigue crack — Structure weakened. High-stress maneuvers (launch, landing, aerocapture, burns >1g) risk Structure failure — roll when attempted. *Crew: reinforcement EVA (4hr) restores full strength.*

Filter clog — Water recycling efficiency -50%. 2-day reserve; then crew dehydration (incapacitation in 24hr, death in 72hr). *Crew: filter replacement (1hr). Spare filters: 2kg, \$500 each; plan 1 per 90 crew-days.*

Flight computer failure — Radiation upset 60%, Component failure (switch to backup; no backup = mission loss) 30%, Total loss (mission loss) 10%. *Hardened systems resist upsets.*

Food contamination — Crew illness, 48hr recovery (no work). Rations -30% from contaminated portion. *Quarantine affected supplies.*

Food spoilage — Rations -20%. *Extend mission by rationing or abort if insufficient.*

Food storage failure — Food spoilage 60%, Food contamination 30%, Total loss (abort required; starvation in 30 days) 10%

Fuel cell failure — Membrane degradation (output -50%) 40%, Cooling loop failure (thermal overload) 25%, Reactant leak (H_2/O_2 loss) 20%, Controller fault (offline) 15%. *Fuel cells produce water — loss affects life support too. Switch to backup power.*

Fuel/core corrosion — Hot hydrogen erodes / cracks NTR fuel elements (NERVA-class life limit). Isp -15%; remaining hot-fire life halved (track remaining burn-minutes at current throttle; at 0, engine is spent). *Can finish the current burn if life remains. Not repairable. Plan shorter remaining burns or accept lower Isp.*

Grapple failure — Mechanism jam (can't close) 40%, Release failure (can't open) 30%, Misalignment (capture miss) 20%, Structural (grapple destroyed on impact) 10%. *Jam: crew EVA to manually release (2hr). Miss: re-approach and retry.*

Grid erosion — Isp -10%, thrust -8% immediately, then -3% Isp per year thereafter (automatic, no roll). *Ion accel/screen grids worn by charge-exchange ions; electron backstreaming rises. Thruster still usable; recalculate transfer times.*

Guidance/navigation failure — Sensor drift 55%, Star tracker failure 30%, IMU failure 15%. *Ground: upload position fixes via DSN.*

Heater malfunction — Cold zones develop. Propellant lines freeze in 12hr (no engine restart) if not rerouted. *Crew: manual heater bypass (1hr) fixes.*

Heat shield failure (†) — Tile damage 50%, Separation failure 25%, Burn-through 25%. *Entry activation only.*

Igniter failure (†) — Pyro / torch / spark train fails to light the grain or chamber. No ignition this attempt. *Activation only (solid † table). Solid: motor inert — jettison if strap-on / use backup stage; no backup means that stage's Δv is gone.*

Ignition failure — Start sequence fails (chilldown, mixture, spark/torch, or pump spin-up). Engine unavailable for this burn. *Comes from † / start tables, not mid-burn F. Retry once (same burn window if time allows). Success: continue. Fail: switch to backup engine, abort burn, or wait for next transfer window.*

IMU failure — Inertial reference lost. Cannot execute burns until fixed. *Crew: manual star sighting (8hr) restores coarse reference ($\pm 5\text{km}$ accuracy, see Sensor drift* penalties). No backup and no crew: mission loss.**

Instrument failure (generic) — Calibration drift (accuracy -40%, data still usable) 45%, Partial degradation (range or sensitivity halved) 30%, Detector noise (requires 2× integration time) 15%, Complete failure (no data) 10%. *Drift: continue with reduced precision; ground recalibration may recover. Degradation: reduced science/data value but still operational. Noise: usable with longer collection windows. Complete: switch to backup or accept data gap.*

Injector failure — Orifice fouling, faceplate damage, or mixture-ratio maldistribution. Partial blockage (thrust -30%; Isp -5%) 75%, Coking → Combustion instability 25%. *Partial: continue degraded or switch backup if timing-critical. Not repairable in flight.*

Joint failure — Localized structural failure. Affected section cannot bear thrust loads; isolate or jettison if possible. If load-bearing and not isolated: high-stress maneuvers (burns >1g, landing) risk *Structure failure* — roll when attempted.

Lab failure — Equipment/instrument failure (experiment offline, data loss for current run) 40%, *Electronic failure* (control systems, data acquisition) 25%, Environmental control fault (temperature/pressure/atmosphere, experiments invalidated) 15%, Contamination (samples cross-contaminated, restart required) 15%, Catastrophic (fire, chemical release, radiation leak; facility damaged, crew injury possible) 5%. *Offline for repair: 1 day per \$10M facility value. Data loss: current research project delayed by 1-2 weeks (restart experiments). Contamination: quarantine affected samples, restart affected experiments. Catastrophic: Physics Lab may cause Radiation leak, Chemical Plant may cause toxic release.*

Landing gear failure (†) — Deployment jam (gear) 40%, Shock absorber failure 30%, Structural → roll *Structure failure* 20%, Complete collapse (vehicle loss, crew survival 50%) 10%. *Landing activation only.*

Life support failure — Atmospheric control failure 40%, Water recycling failure 30%, Thermal regulation failure 20%, Food storage failure 10%

Line leak — Feed line, flex hose, or fitting weeps propellant. Loss 1%/hour from the affected tank. *Abort recommended if leak cannot be isolated. Isolate that branch and continue on redundant feed if designed. Crew: EVA patch possible (2hr) if accessible.*

Mechanical failure (generic) — Bearing/gear wear (degraded, -50% performance) 40%, Motor burnout (offline) 30%, Jam (stuck, clearable) 20%, Structural break (destroyed) 10%. *Degraded: continue with reduced capacity. Offline: switch to backup or manual operation. Jam: crew 1-2hr to clear.*

Major leak (tank) — Large breach or failed fitting. Tank empties in ~1 hour. *Abort immediately or isolate that tank and continue on remaining propellant if the architecture allows. Usually not patchable in time.*

Micrometeorite damage — Power -20% from panel damage. *No repair. Reduce power draw.*

Minor leak (tank) — Slow tank or fitting weep. Propellant loss 1%/day. Mission may continue with reduced margins; recalculate remaining Δv . Crew: EVA patch possible (4hr) if accessible. Isolate tank if redundant tankage exists.

Mining failure — Drill bit wear (output -50%) 35%, Mechanical failure (excavator) 30%, Hopper jam 20%, Structural (collapsed on hard contact) 15%. Bit wear: replace bits (0.1t, \$50k each). Hopper: crew clear (1hr). Structural usually totals the rig.

Neutralizer failure — Hollow cathode / neutralizer can't emit electrons; beam charges the spacecraft and trips the thruster offline (Hall: cathode is this part). Switch to backup thruster. No backup: recalculate mission with reduced thrust; abort if Δv insufficient.

Nozzle erosion — Throat / extension material thinned by heat and exhaust (ablative char, throat widening, or solid nozzle insulator burn). Thrust -15%, Isp -5%. Engine/motor still usable — recalculate Δv . Not repairable in flight.

Nozzle failure — Nozzle erosion 60%, Cooling failure 30%, Burn-through (chamber/nozzle wall fails → Propellant feed failure) 10%. Liquid engines. Solids: Nozzle erosion 70%, Catastrophic nozzle loss (that motor gone — asymmetric thrust; jettison strap-on if possible, else stage/vehicle compromised) 30%. Skip Cooling — solids have no regen loop.

NTR start failure (†) — Reactor scram 40%, Turbopump failure 35%, Chillover/start abort (no thrust this attempt) 25%. Shop † per cold start only. Chillover abort: retry once after reconditioning (1hr), then offline until ground/crew procedure succeeds.

Nuclear thermal failure (F) — Reactor scram 35%, Turbopump failure 30%, Fuel/core corrosion 25%, Radiation leak 10%. While thrusting / hot. Cold start † → NTR start failure. Distinct from chemical combustion modes.

O₂ generator failure — O₂ production halted. 24hr reserve (48hr if crew rests); then crew incapacitation in 2hr, death in 6hr. Crew: switch to backup generator if available.

Oscillation — Sustained chamber-pressure buzz / pogo-like vibration. Thrust -20%, Isp -5%. Engine still running. Throttle to 50% stabilizes. If run at full thrust: 20% chance per burn of escalating to Shutdown required (engine offline).

One-shot failure (†, generic) — Deployment mechanism jam 50%, Pyrotechnic misfire 30%, Structural failure on activation 20%. Activation only. Backup systems essential. No repair once fired.

Parachute failure (†) — Deployment jam (chute) 50%, Partial deployment 35%, Structural tear 15%. Entry/landing activation. Always carry backup chutes.

Partial deployment — Descent rate +50%. Landing: 30% chance of Structure failure. Deploy backup chute if available.

Power electronics failure — PPU / DCIU / discharge supply trips or fails (overcurrent, EMI latch-up, converter death). Thruster offline. Switch to backup thruster. Crew: repair (2hr) if spares carried (0.01t, \$10k per thruster). No backup and no spares: recalculate mission with reduced thrust.

Pressure leak — Atmosphere escaping at 5%/hour. Patch within 6hr or evacuate compartment. Crew: locate and patch (2hr EVA). Seal compartment if needed.

Propellant contamination — Particles, moisture, or iron-nitrate / corrosion products foul valves or injectors after dormancy. Engine cannot start or restart safely. Almost always a † / start outcome, not mid-burn F. Use backup engine or abort. If start attempted anyway: 50% chance of destructive Combustion instability.

Propellant feed failure — Feed system can't deliver propellant at design rate/pressure. Valve stuck 50%, Line leak 35%, Rupture → Propellant tank failure 15%. Crew: line leak patchable EVA if accessible. Valve/rupture: isolate and use backup engine if available.

Propellant flow issue — Xenon/argon feed restricted (valve partial, pressure regulator, or line freeze/clog). Thrust -40%; thruster still lit. Continue slow or switch to backup if time-critical. Crew: may clear with heater/valve cycle (1hr, 50% success) if accessible.

Propellant tank failure — Minor leak (tank) 60%, Major leak (tank) 30%, Tank rupture 10%

Pump failure (water) — Electric pump offline. Crew: hand-pump operation (2hr/day crew time). Uncrewed: N/A (no life support).

Radiation leak — Crew receives lethal dose in 4hr unshielded. *Crew: shelter in shielded quarters (safe while sheltered, but cannot operate spacecraft); jettison reactor module if possible (2hr EVA, ends leak). No jettison capability: abort to nearest safe destination or crew loss.*

Radiation upset — Computer reboot, 1hr downtime. Spacecraft in safe mode during reboot. *Automatic recovery via watchdog timer.*

Radiator failure (F) — Puncture (coolant leak, thermal capacity -25%) 50%, Pump failure (circulation stopped) 30%, Coating degradation (emissivity -30%) 20%. *While rejecting heat. Deployable radiator first unfurl uses † → Deployment jam (panels), not this table. Puncture: isolate panel. Pump: switch to backup loop. Degradation: reduce power generation to match.*

RCS failure — Valve stuck 50%, Thruster leak 30%, Thruster clog (thrust -50%) 15%, Feed line freeze (use heaters) 5%. *Use alternate thrusters. Full RCS loss: switch to CMG/reaction wheels for attitude control.*

Reaction wheel failure — Bearing wear (vibration, jitter) 45%, Motor failure (wheel offline) 30%, Saturation (must desaturate via RCS) 20%, Control electronics 5%. *Spacecraft typically carries 4 wheels; can operate on 3. Below 3: use RCS for attitude.*

Reactor failure (fission) — Control rod jam (scram, safe shutdown) 40%, Cooling failure (scram required) 30%, Radiation leak 20%, Criticality excursion (meltdown, vehicle loss) 10%. *Fission reactors fail safe by design. Meltdown only from cascading failures.*

Reactor scram — Neutronics / protection system trips the reactor to a safe shutdown (rod insertion, temp/flux limit). No thrust until restarted. *Crew or ground: restart attempt (1hr, 70% success). If restart fails, engine offline permanently. Uncrewed deep-space: treat as offline unless autonomous restart is declared in the design.*

Receiver failure — No uplink from ground. Spacecraft executes pre-programmed sequence only; no course corrections, abort commands, or contingency responses possible. *Uncrewed: mission success depends on pre-launch planning. Crewed: crew assumes manual control.*

Refinery failure — Reactor vessel fouling (output -50%) 35%, Thermal failure (process temp) 25%, Pump/valve failure (offline) 20%, Catalyst poisoning (product contaminated, must replace) 15%, Containment breach (facility evacuate, 5% crew injury) 5%. *Fouling: 1 week downtime to clean. Catalyst: \$1M + 0.5t per reactor.*

Robotic arm failure — Bearing wear (stiff, reduced mobility) 40%, Wrist degradation (weak grip) 30%, Sensor drift (inaccurate placement) 20%, Controller fault (safe mode) 8%, Structural damage (destroyed) 2%. *Components replaceable by EVA (6hr, 15% unit cost). Sensor: recalibrate (4hr). Very robust, designed for on-orbit repair.*

Robotic failure (humanoid/surface) — Locomotion joint failure (immobile, arm functions still work) 30%, *Robotic arm failure* (manipulation) 25%, Dust contamination (mechanisms seize over hours; preventive cleaning resets) 20%, *Seal failure* (vacuum breach, electronics destroyed) 15%, Thermal failure (overheat in sun / freeze in shadow, shutdown) 10%. *Joint failure: retrieve by rover or second robot. Dust: clean every 50 operating hours or F doubles. Seal/thermal: unit likely destroyed.*

Rover failure — Wheel damage (continue at reduced speed, path limited) 45%, Drive motor failure (mobility -50%) 20%, Battery failure 15%, Navigation fault (lost, ground tracking to relocate) 10%, Stuck (crew EVA 2hr to free, or abandon) 8%, Rollover (50% recoverable) 2%. *Wheel damage is gradual and primary failure mode. Severe damage eventually immobilizes rover.*

Safe mode entry — Spacecraft in minimal-power safe state; no maneuvers for 48hr while ground diagnoses and uploads recovery commands. *If critical burn window falls within 48hr: burn missed, recalculate mission.*

Sabatier failure — Repeated shutdown (offline, warm reactor 2 days to restart) 40%, Temperature control fault (offline until repaired) 25%, Catalyst poisoning (output -60%, regenerate 2 days) 20%, Feed clog (output -50%, crew 4hr to clean) 10%, Catalyst fracture (replace, 1 week) 5%. *System prone to repeated failures. Restart attempts 60% successful. Major repairs 20-30% unit cost.*

Seal failure (generic) — Slow leak (atmosphere/fluid loss over hours) 60%, Rapid leak (minutes to evacuate/isolate) 30%, Catastrophic (instant decompression, structural damage) 10%. *Slow: locate and patch (2hr). Rapid: isolate compartment. Catastrophic: loss of affected module, possible crew loss.*

Seal leak — Turbopump shaft / dynamic seal weeping propellant into the pump housing or interpropellant cavity. Escalates → *Propellant feed failure. Shutdown engine immediately to prevent cascade. That engine offline unless a designed seal-drain / restart procedure exists (negotiate; default: offline).*

Sensor drift — Position error +1km/day accumulating. Precision maneuvers (orbital insertion, rendezvous, landing) require correction burn: +1% Δv per 10km accumulated error, or accept 10% miss chance per 10km. *Ground: upload corrections via DSN (resets error). Crew: star fix (4hr, resets error).*

SEP restart failure (†) — *Neutralizer failure 40%, Power electronics failure 40%, Propellant flow issue 20%. Shop † at thruster restart only (e.g. T6). No grid/channel wear-out here — those are burn F.*

SEP thruster failure (Hall) (F) — *Channel erosion 40%, Power electronics failure 30%, Propellant flow issue 15%, Neutralizer failure 15%. While thrusting. Restarts → SEP restart failure. Most failures allow degraded operation. Switch to backup or continue at reduced performance.*

SEP thruster failure (ion) (F) — *Grid erosion 45%, Power electronics failure 25%, Propellant flow issue 15%, Neutralizer failure 15%. While thrusting. Restarts → SEP restart failure. Most failures allow degraded operation. Switch to backup or continue at reduced performance.*

Separation failure (†) — *Partial separation (stuck at one point, tilted) 60%, Complete failure (no separation, mission loss unless backup) 25%, Premature separation (early release, likely vehicle loss) 10%, Structural damage on separation 5%. Activation of sep system only. Partial: attempt separation with thrust (70% success, 30% structure damage). Complete: backup separation system or mission loss. Premature: vehicle loss in most cases. Heat shield context: EVA to clear (3hr) if accessible.*

Shock absorber failure — *Hard landing: 50% chance of Structure failure on touchdown. Brace crew for impact. Inspect vehicle post-landing.*

Simple thruster failure (cold gas) (F) — *Valve stuck 70%, Thruster leak 25%, Thruster nozzle clog 5%. While commanded-on / pulsing. Use alternate thrusters. No catalyst — valve/leak dominate.*

Simple thruster failure (monoprop) (F) — *Valve stuck 50%, Thruster leak 25%, Catalyst bed degradation 15%, Thruster nozzle clog 10%. While commanded-on / pulsing. Use alternate thrusters. Long burns on larger REAs raise thermal/clog share in fiction.*

Software fault — *Watchdog reset 70%, Safe mode entry 30%. Ground: upload patch after recovery.*

Spectrometer failure — *Calibration drift (accuracy -40%) 45%, Wavelength range loss (partial spectrum only) 30%, Detector noise (requires 2× integration time) 15%, Complete failure (no readings) 10%. Drift: continue with less precise identification. Range loss: can still identify some materials. Noise: longer data collection time. Complete: switch to backup or rely on cameras only.*

Sail deploy failure (†) — *Boom/membrane jam (sail useless) 50%, Tear on unfurl (area -30%, thrust -30%) 30%, Incomplete deploy (thrust -50%) 20%. Shop † at sail deploy only. Jam: no photon thrust. Tear/incomplete: recalculate transfer; no repair in flight.*

Solar panel failure (F) — *Cell degradation (radiation, thermal cycling; output -5% per event) 60%, String failure (ESD or manufacturing; one section offline, output -10–25%) 25%, Wiring fault → Electrical fire 10%, Connector corrosion (intermittent output loss, repairable) 5%. While generating. First deploy † → Deployment jam (panels). Tracking motors use their own F → Actuator failure* only while driving. Micrometeorite/debris: environmental F (Astrodata)*.*

Solar sail failure (F) — *Micrometeoroid / tear (area -10%, thrust -10%) 50%, Attitude/sail-control loss (can't steer sail; thrust direction fixed or zero useful) 30%, Slow optical degradation (thrust -5% permanent) 20%. Cruise after successful deploy. Deploy † → Sail deploy failure.*

Solid motor failure (†) — *Casing breach 45%, Nozzle failure 45%, Igniter failure 10%. Per-fire † — covers light through burnout (solids have no separate burn F). Not repairable. QC is the only defense.*

Spin gravity failure — *Bearing degradation (vibration, noise, degrade 0.3g only) 35%, Rotating seal leak (atmosphere loss 2%/day, isolate section or repair) 30%, Gyroscopic imbalance (tumbling risk, must despin) 15%, Motor/controller fault (can't spin up/down) 12%, Catastrophic bearing seizure (sudden stop, crew injury 50%, structure damage) 8%.*

Bearing degradation: continue at lower RPM to reduce stress. Seal leak: crew EVA repair (4hr) with spares (5% unit cost). Imbalance: emergency despin via RCS, rebalance mass distribution. Motor fault: manual emergency brake (crew) or ground diagnosis + patch upload. Catastrophic: immediate despin, potential crew injury (broken bones from sudden decel), inspect structure for damage before restart.

Star tracker failure — Navigation accuracy $\pm 10\text{km}$ instead of $\pm 1\text{km}$. Precision maneuvers require $+1\%$ Δv correction, or accept 10% miss chance. *Crew: manual star sighting (8hr) restores $\pm 5\text{km}$ (halves penalty).*

Storable engine failure (F) — Valve stuck 45%, Chamber degradation 35%, Nozzle erosion 20%. Mid-burn / while firing. Degraded chamber/nozzle often acceptable — finish at lower Isp. Start/relight $\nrightarrow$ Storable start failure.

Storable start failure ($\nrightarrow$) — Valve stuck 55%, Propellant contamination 30%, Ignition failure 15%. *Shop $\nrightarrow$ at ignition/activation after dormancy. Never from burn-day F.*

Structural tear — Descent rate $+100\%$. Hard landing: 70% chance of Structure failure. Deploy backup chute. Brace for impact.

Structure failure — Fatigue crack 50%, Joint failure 30%, Major structural failure 20%. *Major: if under thrust or atmospheric load, vehicle loss; otherwise, abort mission (no further maneuvers possible).*

Surface habitat failure — Life support failure 40%, Seal failure (hull) 25%, Thermal regulation failure 20%, Power distribution fault 10%, Structural (meteorite/collapse) 5%. *Crew: evacuate to backup shelter or vehicle. Repair times vary by damage — negotiate.*

Tank rupture — Propellant tank bursts (overpressure, pump debris, structural failure). Vehicle loss. *Catastrophic; no repair. Isolate other tanks only if the design has independent tankage and the rupture didn't cascade.*

Tether failure — Fatigue snap (tether parts at weak point) 40%, Micrometeorite cut 25%, Deployment tangle (jammed, must cut loose) 20%, Connection failure (fitting breaks) 15%. *Snap: payload lost on ballistic trajectory — calculate where it goes. Tangle: lose tether but save payload.*

Tether deployer failure — Motor jam (tether stuck) 50%, Brake failure (uncontrolled payout) 30%, Reel tangle (cut loose) 20%. *Motor jam: crew manual crank (2hr) if accessible. Uncontrolled payout may overstress tether $\rightarrow$ snap.*

Thermal failure (generic) — Sensor fault (false readings, manual override required) 40%, Heater element burnout (cold spots develop) 30%, Coolant pump failure (hot spots develop) 25%, Runaway (temp excursion, equipment damage) 5%. *Manual override: crew monitors and adjusts. Burnout/pump: redistribute loads, rotate spacecraft.*

Thermal regulation failure — Heater malfunction 50%, Cooling loop failure 40%, Total failure (crew death in 6hr; evacuate to docked vehicle or surface shelter if available) 10%. *Rotate spacecraft for passive thermal control.*

Thruster leak — Seat, fitting, or nozzle joint leaks propellant/pressurant overboard. Loss 5%/hour from affected thruster's tank (or shared tank if common). *Isolate feed line to that thruster. Use alternate thrusters. Crew EVA patch (2hr) only if the leak is on an accessible external fitting — seat leaks usually need thruster swap on ground.*

Thruster nozzle clog — Orifice / catalyst-screen / nozzle throat restricted by particles or decomposition products. Thrust -50% from affected thruster. *Use alternate thrusters for attitude control. Not clearable in flight for most REAs.*

Tile damage — Localized heating during reentry. Vehicle survives if structure was undamaged; if structure had prior Fatigue crack or Joint failure, 50% chance of vehicle loss. *Crew: tile repair EVA (4hr) if spotted pre-reentry prevents damage entirely.*

Tracking failure — Solar panels not pointing at sun. Power -30%. *Crew: manual pointing (1hr/day) restores full power. Fixed panels unaffected.*

Transfer pump failure — Seal failure (propellant leak during transfer, 10% loss) 40%, Motor burnout (offline) 30%, Valve stuck (can't start/stop transfer) 20%, Contamination (cross-contamination of tanks) 10%. *Seal: abort transfer, patch. Motor: manual transfer possible but slow ($10\times$ time). Contamination: affected tank unusable.*

Transmitter failure — Effective range halved (beyond Mars orbit without DSN boost: no contact), data rate -50% (science return halved). *Ground: DSN (Deep Space Network — Earth's large radio dishes) boost restores range but not data rate.*

Turbopump failure — High-speed pump/turbine can't deliver design flow. Bearing wear (thrust -25%; vibration) 40%, Seal leak 30%, Seizure (immediate shutdown; that engine offline) 20%, Catastrophic (pump disintegration / fire path) → Propellant tank failure 10%. Bearing wear: continue degraded or shut down to save the engine for later. Not repairable in flight.

Valve stuck — Propellant or pressurant valve won't move (contamination, cold weld, actuator fail). Partial (thrust -50%) 33%, Stuck-off (no flow — that engine/thruster shutdown) 33%, Stuck-on (runaway thrust until upstream line isolated) 34%. Isolate if stuck-on. Switch to backup engine or alternate thrusters. ACS blocks: lose that thruster; keep flying on the rest of the pod.

Watchdog reset — Computer reboots, 1hr downtime. Normal recovery. Monitor for recurring faults.

Water contamination — Water supply unsafe. Use reserves (5-day supply); then dehydration (incapacitation in 24hr, death in 72hr). Crew: emergency filter (4hr) salvages 50% of contaminated supply.

Water recycling failure — Filter clog 50%, Pump failure (water) 35%, Water contamination 15%. Crew: filter replacement if spares available.

Whipple shield damage — Minor penetration (shield holed, no internal damage) 70%, Shield layer loss (protection -50%, vulnerable to future impacts) 20%, Penetration to pressure hull → Pressure leak 8%, Critical system damage (roll appropriate system failure) 2%. Minor: continue mission. Layer loss: reduced protection against future impacts. Hull penetration: immediate leak response. Critical: cascade failure to affected system.

10. Random Tables

15	39	59	91	50	01	41	77	69	46	94	34	35	16	71	03	73	85	56	72	05	31	18	74	17	50	88	03	25	66
65	58	84	04	37	43	82	91	58	40	63	59	25	14	67	08	32	29	31	79	82	42	19	77	35	72	21	30	28	52
20	91	60	96	21	56	41	41	93	08	31	08	78	91	09	85	86	57	50	67	12	56	12	08	30	68	77	43	80	79
11	04	44	35	45	49	17	03	80	91	02	81	38	78	70	72	38	36	54	04	36	51	14	87	51	08	97	11	77	49
38	50	09	57	54	03	46	92	54	64	26	54	50	03	66	26	05	92	37	37	14	90	35	46	88	06	29	39	24	36
51	45	73	76	81	01	86	66	90	94	74	80	74	67	34	90	17	92	44	49	06	84	02	66	27	19	94	91	35	21
67	09	42	91	45	15	05	85	44	87	37	56	85	93	25	67	87	33	19	82	51	57	37	97	45	45	16	07	11	07
65	93	33	67	30	66	41	17	05	34	09	11	46	22	75	52	22	16	37	85	60	08	83	39	78	88	51	01	08	18
49	41	77	63	13	77	77	56	99	66	85	97	26	33	16	42	16	68	95	57	14	57	44	05	52	83	59	72	06	20
70	98	24	40	99	42	14	97	44	57	81	88	99	77	29	91	84	18	56	75	59	82	91	18	90	07	91	40	23	17
58	40	61	15	29	19	47	91	31	03	37	95	90	01	44	56	24	96	05	32	93	48	13	21	10	33	85	41	67	50
82	43	78	90	17	81	53	11	64	20	83	71	08	50	64	44	91	30	24	57	98	53	69	10	77	90	88	84	94	48
01	87	42	98	76	57	75	12	79	61	99	76	96	37	52	39	68	65	77	37	45	37	96	23	93	38	21	59	42	75
05	32	70	94	57	70	28	29	22	26	63	31	02	91	58	18	88	63	55	26	19	57	91	37	48	12	33	16	18	06
05	21	11	15	20	51	80	07	61	41	48	15	85	25	51	04	85	71	87	97	41	74	72	70	30	07	60	47	84	86
30	82	69	21	33	99	79	65	99	57	41	58	72	57	75	49	17	71	61	91	19	25	16	52	88	53	63	05	82	08
15	68	11	42	61	41	86	41	89	17	71	19	10	99	23	66	85	30	96	74	99	32	56	04	83	37	34	32	85	02
63	49	81	04	68	05	05	83	31	75	67	94	52	93	65	70	59	80	30	78	25	66	38	25	67	61	54	83	22	63
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09 21 52 10 01 63 74 37 17 51	67 55 37 96 46 14 85 65 29 97	79 68 79 84 68 68 60 19 24 13
87 46 29 69 78 90 23 31 47 56	23 99 11 05 40 48 70 34 94 02	68 19 40 83 61 18 74 36 91 94
07 84 65 39 17 58 50 49 11 62	34 79 91 10 58 61 63 97 52 06	56 48 16 19 37 03 67 75 96 63
76 23 39 37 76 82 80 52 92 02	54 12 20 05 30 55 75 85 32 76	35 30 75 66 03 49 02 39 58 85
03 92 38 67 52 02 32 27 93 02	16 10 03 55 07 83 87 98 14 31	67 73 75 65 48 39 05 87 14 45
82 89 23 78 63 41 13 67 25 80	07 71 86 73 51 18 96 84 99 63	30 95 56 32 94 19 95 02 41 89
15 18 30 31 64 41 44 65 06 86	69 88 64 45 31 49 40 50 39 05	53 54 88 81 91 98 50 60 72 91
48 92 50 69 19 72 84 49 14 68	34 05 27 02 91 82 71 67 51 39	83 14 99 85 83 74 89 55 39 96
75 06 60 52 50 14 51 05 34 05	83 10 03 44 39 20 77 69 36 86	17 81 21 65 18 58 53 63 88 57
73 06 81 31 97 65 37 84 60 31	05 06 34 52 50 53 50 87 69 82	55 99 77 96 32 03 78 43 32 43
18 96 54 10 32 83 94 58 52 03	88 96 36 44 35 93 46 14 37 83	38 82 32 28 50 12 78 67 59 21
96 18 29 41 31 09 55 39 24 92	73 71 93 44 59 84 88 68 32 10	91 36 37 54 29 71 12 16 47 83
80 70 58 52 68 78 45 20 93 13	28 35 24 77 59 59 37 55 17 40	43 89 59 16 13 84 82 71 58 82
30 61 36 64 95 23 30 49 89 40	60 86 15 86 15 19 78 89 40 02	35 29 81 59 07 53 16 10 44 16
85 93 97 40 09 34 77 22 89 17	30 62 60 13 11 83 61 91 56 22	81 81 34 92 80 30 73 53 75 45
35 15 71 75 75 03 59 37 25 82	01 01 18 16 36 47 51 25 70 75	95 05 63 61 47 25 47 50 76 99
09 20 23 53 86 03 36 20 72 48	81 58 84 06 67 38 50 63 45 70	19 45 98 73 82 66 52 72 30 12
16 37 83 71 30 44 02 35 86 51	72 81 72 47 37 15 15 83 40 07	95 31 31 88 60 40 31 40 64 10
09 90 16 78 68 39 94 50 87 04	50 77 25 31 15 08 45 41 19 29	62 37 74 38 98 57 48 60 62 96
06 95 72 21 77 19 59 35 67 61	52 80 63 38 80 42 04 13 77 49	43 34 40 98 14 03 25 52 05 33
17 84 29 54 31 76 72 95 66 77	76 59 84 94 59 82 22 37 18 74	08 38 61 90 82 27 99 59 49 34
10 64 20 56 85 91 04 92 99 26	32 18 14 40 80 50 54 79 08 47	85 77 62 08 05 47 97 70 51 13
22 02 89 59 20 73 57 20 52 24	22 86 83 96 30 70 40 34 65 73	10 54 98 34 99 34 11 57 29 25
57 27 90 70 85 61 12 84 26 40	34 22 31 09 72 94 58 22 63 19	39 86 26 37 10 61 84 02 53 56
88 11 77 50 92 44 81 68 35 08	75 31 78 59 95 45 86 07 51 12	63 01 27 92 30 06 71 54 19 15
51 90 22 15 08 69 81 97 98 48	27 19 39 74 54 06 82 72 32 52	98 15 51 98 84 51 20 02 37 75
20 12 53 56 97 83 72 22 25 57	44 84 71 72 29 72 59 07 69 93	38 30 23 86 96 79 24 41 08 42
36 29 90 17 11 48 94 99 52 79	53 51 46 23 15 70 95 63 80 50	96 99 88 72 19 67 18 13 99 87
93 20 09 13 02 61 72 75 22 72	30 04 49 23 96 40 65 43 75 65	64 22 67 70 32 42 70 82 82 95
01 12 06 41 77 34 46 24 93 13	37 13 66 39 28 92 82 21 84 48	60 83 18 95 73 54 95 82 97 32
97 37 04 06 57 95 90 77 63 64	12 34 80 14 24 38 24 79 98 67	29 22 51 01 86 61 86 31 90 40
05 28 41 63 14 86 64 95 61 31	24 25 19 34 61 84 57 71 21 42	81 45 23 83 84 39 97 05 38 72
03 46 71 46 71 39 01 52 27 79	10 30 72 91 88 38 87 93 56 44	15 21 63 79 81 37 91 52 72 94
74 57 90 93 41 72 70 35 11 41	94 34 22 15 88 11 25 08 17 39	16 75 68 45 70 86 55 55 89 06
20 56 02 15 89 11 13 32 15 76	07 79 86 41 92 18 83 94 31 38	47 13 10 65 52 82 76 85 51 75
92 81 25 14 30 49 19 31 27 11	39 71 58 62 90 35 70 89 39 53	75 55 68 35 05 58 41 01 15 42
76 84 51 94 78 24 94 01 32 85	13 62 08 34 85 13 63 12 03 79	87 14 37 42 99 05 77 90 73 25
74 76 40 01 79 22 19 58 72 39	09 52 37 49 47 96 19 79 67 08	30 37 03 21 83 37 55 63 10 02
25 33 63 71 53 94 96 21 23 07	95 81 99 69 82 15 76 33 62 67	46 55 80 01 11 65 47 54 22 29
21 55 89 61 90 96 65 62 17 78	56 18 15 66 73 40 34 98 36 30	55 32 16 33 11 42 47 05 74 01
22 10 28 04 86 83 67 91 61 07	25 73 55 14 72 88 55 68 64 96	77 36 37 91 12 59 34 45 13 56
92 79 64 80 86 48 80 32 49 04	20 89 83 49 77 75 07 66 75 12	16 65 55 79 16 46 47 10 32 71
58 87 02 51 40 43 22 10 17 65	67 15 90 48 48 74 65 81 89 37	84 41 98 66 19 13 10 40 11 44
69 87 30 79 87 24 89 85 99 54	63 32 19 24 65 44 82 01 31 90	17 10 29 61 93 51 43 25 68 39
97 57 03 39 29 81 97 93 55 56	82 65 27 01 74 20 96 66 42 75	58 11 41 25 19 59 51 64 92 82
71 89 75 43 18 95 29 17 84 50	27 86 46 35 92 46 58 71 90 59	55 35 93 25 39 29 33 54 36 26
38 20 68 39 76 41 54 02 97 04	80 77 01 71 37 11 61 98 46 85	92 54 10 57 94 13 15 21 21 46
06 85 85 56 13 56 88 78 75 20	24 05 81 61 11 64 64 46 84 02	74 30 83 76 95 24 87 17 41 68
09 50 83 75 13 62 78 83 54 70	30 10 70 55 06 26 36 88 04 66	26 16 73 07 75 16 39 69 11 29
10 46 85 52 18 27 23 23 88 50	26 59 53 13 50 19 09 21 02 18	66 58 47 14 29 98 82 75 57 78
25 57 57 06 69 30 79 26 80 52	44 08 17 47 80 85 39 65 88 05	13 44 83 94 76 85 12 02 04 35
48 28 84 21 92 94 70 26 53 37	66 52 89 40 20 97 35 38 61 59	94 23 61 26 14 47 77 61 30 90
27 63 80 39 24 85 78 34 17 69	03 99 34 54 19 54 24 36 96 44	97 10 90 57 76 81 40 65 22 90
34 21 05 07 54 68 49 85 15 34	29 01 86 06 66 77 25 34 99 70	13 17 64 84 77 25 53 25 37 35
87 84 93 27 82 19 90 96 59 23	28 33 99 28 05 92 11 41 45 52	18 61 08 84 04 26 68 05 09 76
66 83 48 25 47 51 56 03 25 12	82 24 77 43 91 51 33 60 18 05	11 99 60 21 49 70 60 17 58 68
61 52 49 21 41 08 95 21 31 97	62 22 53 12 39 26 54 81 09 59	71 74 19 80 52 93 61 19 16 32
88 89 35 98 15 92 14 57 96 13	04 81 65 13 29 33 71 50 48 89	14 89 11 16 49 96 75 25 66 46

Part II

Exercises

11. Falcon 9 Build Exercise

Goal: Design a two-stage kerolox launcher from Shop parts that puts **22,800 kg in LEO** (Falcon 9 expendable class). Size it, cost it, check its risk, then compare against the real vehicle.

11.1 GIVEN

Item	Shop
Propellant	Kerolox — Isp 310s $\rightarrow v_e = 3.041$ km/s; \$3/kg
Engine	Merlin 1D — 845 kN; 470 kg; \$1M; 0.1%† per ignition + F:120. Captive [SpaceX]
Tanks	\$100k/t; $s = 0.12$ (tank mass $\div$ propellant mass)
Separation	Separation System — \$100k; 5 kg; 0.2%†
Avionics	\$0.5M
Δv to LEO	9.2 km/s (Physics Handbook: Ascent, losses included)

Sizing (Physics Handbook: Rocket Physics):

$$R = e^{\Delta v / v_e} \quad m_{\text{wet}} = \frac{m_{\text{fixed}} R}{1 + s - sR} \quad m_{\text{prop}} = \frac{m_{\text{wet}} - m_{\text{fixed}}}{1 + s}$$

Thrust targets: liftoff TWR ≥ 1.3 ; upper stage one engine.

11.2 STEP 1 — SPLIT THE Δv

Both stages burn kerolox, so the split sets the size. Size the whole vehicle for several splits (Steps 2–3) and keep the lightest:

Upper-stage Δv	S1 engines	S2 wet	Liftoff mass
3.7 km/s	30	110 t	1,951 t
4.2 km/s	25	144 t	1,618 t
4.7 km/s	24	196 t	1,534 t
5.2 km/s	25	282 t	1,625 t
5.7 km/s	30	449 t	1,975 t

Use **S2 = 4.7 km/s**, **S1 = 4.5 km/s**.

11.3 STEP 2 — UPPER STAGE

$m_{\text{fixed}} = \text{payload} + \text{one Merlin} = 22,800 + 470 = \mathbf{23,270 \text{ kg}}$.

$$R = e^{4.7/3.041} = 4.690 \quad 1 + s - sR = 1.12 - 0.563 = 0.557$$

$$m_{\text{wet}} = \frac{23,270 \times 4.690}{0.557} = \mathbf{195,890 \text{ kg}}$$

- Propellant: $(195,890 - 23,270)/1.12 = \mathbf{154,125 \text{ kg}}$
- Tanks: $0.12 \times 154,125 = \mathbf{18,495 \text{ kg}}$
- TWR at ignition: $845,000/(195,890 \times 9.81) = \mathbf{0.44}$

11.4 STEP 3 — BOOSTER

m_{fixed} = upper stage + n Merlins. Engine count depends on liftoff mass, so iterate: guess n , size, recount $n = \lceil 1.3 m_{\text{wet}} g / 845 \text{ kN} \rceil$. It settles at $n = 24$.

$$m_{\text{fixed}} = 195,890 + 24 \times 470 = 207,170 \text{ kg.}$$

$$R = e^{4.5/3.041} = 4.392 \quad 1.12 - 0.527 = 0.593$$

$$m_{\text{wet}} = \frac{207,170 \times 4.392}{0.593} = \mathbf{1,534,320 \text{ kg}}$$

- Propellant: **1,184,956 kg**
- Tanks: **142,195 kg**
- Liftoff TWR: $24 \times 845,000 / (1,534,320 \times 9.81) = \mathbf{1.35}$

Burn times (Merlin mass flow = $845,000/3,041 = 277.9 \text{ kg/s}$):

- S1: $1,184,956 / (24 \times 277.9) = \mathbf{178 \text{ s}}$
- S2: $154,125 / 277.9 = \mathbf{555 \text{ s}}$

11.5 STEP 4 — COST

Item	Quantity	Cost
Merlin 1D	25	\$25.0M
Propellant	1,339 t	\$4.0M
Tanks	160.7 t	\$16.1M
Avionics + separation	—	\$0.6M
Hardware		\$45.7M

Per launch = hardware $\times 1.2 \times$ volume multiplier (Shop: Volume Discounts). Ops is a standing program, not a per-flight MC fee: Hot $\ell = 3$ (3 Ground, \$750k/yr) + LEO duty contact \$500k/yr; each flight needs peak $p = 6$ for 2 weeks (Shop: Operations).

Units built	Multiplier	Per launch (hardware)
1–5	5.0×	\$274M
6–20	3.5×	\$192M
21–100	2.5×	\$137M

Units built	Multiplier	Per launch (hardware)
101–200	1.5×	\$82M
201+	1.0×	\$55M

Procurement: Merlin is Captive, so external supply is 0. Developing your own Merlin-class engine: $D_{\text{dev}} = 10 \times \$1\text{M} = \$10\text{M}$ (T5), minimum 1 year, plus lab rent or ops.

Buy instead: M-class service is \$70M for 22 t to LEO, †Launch 1%.

11.6 STEP 5 – RISK

On the pad. All 24 S1 engines light while the vehicle is held down: $1 - 0.999^{24} = 2.4\%$ chance at least one fails its † roll. The vehicle is not released; resolve *Chemical ignition failure* and recycle the count (a scrub, not a loss).

In flight

Event	Type	Probability
S1 burn, first engine-out	Bank rate $24 \times 120/\text{kd}$ over 178 s (2.06×10^{-6} kd)	$E = 0.0059 \rightarrow 0.6\%$
S1 burn, stage loss (3 of 24 out)	$p = 2.5 \times 10^{-4}$ per engine; $\binom{24}{3}p^3$	3×10^{-8}
Stage separation	0.2%†	0.2%
S2 ignition	0.1%†	0.1%
S2 burn	$120 \times 6.4 \times 10^{-6}$ kd	$E = 0.0008 \rightarrow 0.08\%$

Loss of mission $\approx 0.4\%$, almost all of it from separation, S2 ignition, and the single-engine S2 burn. A first-stage engine-out is roughly a 1-in-170 event; roll *Chemical engine failure* (F) for the failed engine and fly on 23.

11.7 STEP 6 – CHECK AGAINST THE REAL FALCON 9

Real Falcon 9: **549 t** at liftoff, **9** Merlins. This design: **1,534 t**, 24 Merlins. Two parameters account for the difference:

- **Upper-stage Isp.** Falcon’s second stage runs a vacuum-nozzle Merlin at about **348 s**; the Shop’s 310s is the kerolox class value.
- **Tank fraction.** Falcon’s tanks and stage structure weigh about **5%** of their propellant; the Shop’s 12% is a general-purpose value.

Re-run Steps 1–3 with each change (best split each time):

Case	Liftoff mass	S1 engines
Shop values	1,534 t	24
S2 Isp 348s	1,082 t	17
Tanks $s = 0.05$	737 t	12
Both	573 t	9
Real Falcon 9	549 t	9

With both changes the method lands within 5% of the real vehicle. Because the stage mass sits in the $1 + s - sR$ denominator, structure fraction moves the result more than any other input. That is why the real Falcon 9 is built around light tanks.

In game terms both changes are improvement programs (Shop: Improving Existing Tech), with P negotiated:

- Vacuum nozzle, Isp 310 $\rightarrow$ 348s (+12%): $D_{\text{dev}} \times (12/25)^2 = 0.23 \times \$10\text{M} \approx \$2.3\text{M}$.
- Tanks at 5% instead of 12% (-58%): $D_{\text{dev}} \times (58/25)^2 \approx 5.4 \times D_{\text{dev}}$ of the tank line.

11.8 INSIGHTS

1. **Split matters** — Equal-Isp stages want roughly equal Δv ; this vehicle is lightest near 4.7 / 4.5 km/s.
2. **Structure fraction dominates** — Going from 12% to 5% tanks shrinks the vehicle about 2 $\times$; the vacuum nozzle is worth about 1.4 $\times$.
3. **Engine-out** makes S1 engine failures survivable; the risk sits in single-string events (separation, S2 light, S2 burn).
4. **Many engines, many lights** — 24 ignitions make pad scrubs common (2.4%) even though each engine is reliable.
5. **Buy vs build** — At 201+ units the built vehicle costs about \$65M per launch against \$70M for M-class, before D_{dev} , factories, and lead times.

12. Atlas V Build Exercise

Goal: Design an Atlas V 401-class launcher from Shop parts that puts **9,800 kg in LEO**: a kerolox booster with one RD-180 and a hydrolox upper stage with one RL-10. Size it, cost it, check its risk, then compare against the real vehicle.

12.1 GIVEN

Item	Shop
Booster engine	RD-180 – 4,150 kN; 5,480 kg; \$15M; 0.5%† per ignition + F:1500. Single-source [Energomash, RU]
Booster propellant	Kerolox – Isp 310s $\rightarrow v_e = 3.041$ km/s; \$3/kg
Upper-stage engine	RL-10 – 110 kN; 190 kg; \$15M; 0.25%† per ignition + F:60. Single-source [Aerojet]
Upper-stage propellant	Hydrolox – Isp 450s $\rightarrow v_e = 4.415$ km/s; \$5/kg
Tanks	\$100k/t; $s = 0.12$ (tank mass $\div$ propellant mass)
Separation	Separation System – \$100k; 0.2%†
Avionics	\$0.5M
Δv to LEO	9.2 km/s (Physics Handbook: Ascent, losses included)

Sizing (Physics Handbook: Rocket Physics):

$$R = e^{\Delta v/v_e} \quad m_{\text{wet}} = \frac{m_{\text{fixed}} R}{1 + s - sR} \quad m_{\text{prop}} = \frac{m_{\text{wet}} - m_{\text{fixed}}}{1 + s}$$

Thrust limits: one engine per stage, so thrust is fixed and the split must respect it.

- Liftoff TWR ≥ 1.2
- Upper-stage TWR ≥ 0.3 . Below that, the long burn loses too much to gravity (Physics Handbook: Gravity loss).

12.2 STEP 1 – SPLIT THE Δv

Hydrolox is the more efficient stage, so loading it with more Δv shrinks the vehicle until the thrust limits bind:

Upper-stage Δv	S2 wet	Liftoff mass	Liftoff TWR	S2 TWR
4.0 km/s	30.0 t	430 t	0.98 ×	0.37
4.3 km/s	33.0 t	371 t	1.14 ×	0.34
4.5 km/s	35.2 t	342 t	1.24	0.32
4.6 km/s	36.3 t	330 t	1.28	0.31
4.8 km/s	38.8 t	308 t	1.37	0.29 ×
5.0 km/s	41.5 t	291 t	1.45	0.27 ×

Use **S2 = 4.6 km/s**, **S1 = 4.6 km/s**.

12.3 STEP 2 — UPPER STAGE

$m_{\text{fixed}} = \text{payload} + \text{RL-10} = 9,800 + 190 = \mathbf{9,990 \text{ kg}}$.

$$R = e^{4.6/4.415} = 2.835 \quad 1 + s - sR = 1.12 - 0.340 = 0.780$$

$$m_{\text{wet}} = \frac{9,990 \times 2.835}{0.780} = \mathbf{36,318 \text{ kg}}$$

- Propellant: $(36,318 - 9,990)/1.12 = \mathbf{23,507 \text{ kg}}$
- Tanks: **2,821 kg**
- TWR: $110,000/(36,318 \times 9.81) = \mathbf{0.31}$

12.4 STEP 3 — BOOSTER

$m_{\text{fixed}} = \text{upper stage} + \text{RD-180} = 36,318 + 5,480 = \mathbf{41,798 \text{ kg}}$.

$$R = e^{4.6/3.041} = 4.539 \quad 1.12 - 0.545 = 0.575$$

$$m_{\text{wet}} = \frac{41,798 \times 4.539}{0.575} = \mathbf{329,705 \text{ kg}}$$

- Propellant: **257,059 kg**
- Tanks: **30,847 kg**
- Liftoff TWR: $4,150,000/(329,705 \times 9.81) = \mathbf{1.28}$

Burn times (mass flow = thrust $\div$ v_e):

- S1: RD-180 at $4,150,000/3,041 = 1,365 \text{ kg/s} \rightarrow 257,059/1,365 = \mathbf{188 \text{ s}}$
- S2: RL-10 at $110,000/4,415 = 24.9 \text{ kg/s} \rightarrow 23,507/24.9 = \mathbf{943 \text{ s}}$

12.5 STEP 4 — COST

Item	Quantity	Cost
RD-180	1	\$15.0M
RL-10	1	\$15.0M
Kerolox	257.1 t	\$0.8M
Hydrolox	23.5 t	\$0.1M
Tanks	33.7 t	\$3.4M
Avionics + separation	—	\$0.6M
Hardware		\$34.9M

Per launch = hardware $\times 1.2 \times$ volume multiplier. Ops is a standing program, not a per-flight MC fee: Hot $\ell = 3$ (3 Ground, \$750k/yr) + LEO duty contact \$500k/yr; each flight needs peak $p = 6$ for 2 weeks (Shop: Operations).

Units built	Multiplier	Per launch (hardware)
1–5	5.0×	\$209M
6–20	3.5×	\$147M
21–100	2.5×	\$105M
101–200	1.5×	\$63M
201+	1.0×	\$42M

Engines are 86% of the hardware bill; propellant is under 3%.

Procurement (Shop: Procurement Flags):

- **RD-180** — N/yr <1 → 24–36 month lead time, +6 months single-source. Spot price 1.0–1.5× list. Supply disruption: d100 ≤ 15 each year (sanction-vulnerable source) → 12-month outage.
- **RL-10** — N/yr 10 → 3–12 months, +6 months single-source. Disruption d100 ≤ 5 per year.

A long-term contract halves the disruption roll; qualifying a second booster engine costs its full D_{dev} .

12.6 STEP 5 — RISK

On the pad. The RD-180 lights before release: a 0.5%† failure is a scrub. Resolve *Chemical ignition failure* and recycle.

In flight. Both stages are single-engine, so there is no engine-out.

Event	Type	Probability
RD-180 burn	1500 × 188 s (2.18 × 10 ⁻⁶ kd)	$E = 0.0033 \rightarrow 0.33\%$
Stage separation	0.2%†	0.2%
RL-10 ignition	0.25%†	0.25%
RL-10 burn	60 × 943 s (1.09 × 10 ⁻⁵ kd)	$E = 0.0007 \rightarrow 0.07\%$

Total ≈ 0.85%. The largest line is the booster’s burn: a high catalog F with no backup engine. Not every RD-180 failure is a loss. *Chemical engine failure* includes oscillation and partial-thrust modes; with only 1.28 TWR, though, a thrust loss early in the burn usually means the vehicle falls back.

12.7 STEP 6 — CHECK AGAINST THE REAL ATLAS V 401

	This design	Real Atlas V 401
Liftoff mass	330 t	334 t
Booster propellant	257 t	284 t
Upper-stage propellant	23.5 t	20.8 t
Upper-stage burn	943 s	~840 s
Price	\$115M at 21–100 units	~\$110M (≈100 flights)

Shop values reproduce this vehicle closely. The hydrolox upper stage carries half the Δv at 450s, which keeps both stages well clear of the $1 + s - sR \rightarrow 0$ limit. Compare the Falcon 9 exercise, where two kerolox stages at Shop tank fractions grow to nearly 3× the real vehicle.

12.8 INSIGHTS

1. **High-Isp upper stage** — Putting half the Δv on hydrolox makes a single-engine booster possible.
2. **Thrust bounds the split** — With one engine per stage, the lightest vehicle sits where liftoff TWR and upper-stage TWR limits meet.
3. **Single-string risk** — No engine-out means the booster's burn F shows up directly in mission risk; compare Falcon 9's bank.
4. **Engines are the cost** — Two engines are \$30M of a \$35M vehicle. Volume discounts and supply contracts matter more than propellant or tanks.

12.9 EXTENSION

Atlas V 551-class (18,850 kg to LEO): add strap-on solids. Shop **GEM-63**: 1,650 kN, 49 t wet, \$8M, 0.25%†. Size how many you need for liftoff TWR ≥ 1.2 with the heavier upper stage. Each motor is one *Solid motor failure* roll covering light through burnout.

13. Ceres Mission Exercise

Goal: Put a **500 kg orbiter** in a 100 km orbit around Ceres and land a **200 kg science package** on the surface. Choose the propulsion, size the spacecraft, price the mission.

Ceres: the largest body in the asteroid belt (2.77 AU), a dwarf planet with water ice, clays, and organics.

13.1 GIVEN

Item	Value
Earth orbit	1.00 AU
Ceres orbit	2.77 AU; period 1,680 days
Ceres	radius 476 km; $\mu = 63 \text{ km}^3/\text{s}^2$
Sun	$\mu_{\odot} = 1.327 \times 10^{11} \text{ km}^3/\text{s}^2$; 1 AU = $1.496 \times 10^8 \text{ km}$
Orbiter	500 kg; \$10M (given)
Lander science package	200 kg; \$5M (given)

13.2 STEP 1 – IMPULSIVE Δv (CHEMICAL)

Hohmann transfer (Physics Handbook: Transfers):

$$a_t = \frac{1.496 + 4.144}{2} \times 10^8 = 2.820 \times 10^8 \text{ km}$$

Quantity	Value
$v_{\text{Earth}} = \sqrt{\mu_{\odot}/r_1}$	29.78 km/s
$v_{t,1}$ (perihelion of transfer)	36.1 km/s
v_{∞} at departure	6.3 km/s
v_{Ceres}	17.89 km/s
$v_{t,2}$ (aphelion of transfer)	13.0 km/s
v_{∞} at arrival	4.9 km/s

Departure from 200 km LEO: $v_{\text{orbit}} = 7.79 \text{ km/s}$; $v_{\text{hyp}} = \sqrt{6.3^2 + 2 \times 398,600/6,571} = 12.70 \text{ km/s} \rightarrow \text{TCI} = 4.91 \text{ km/s}$.

Capture at 100 km ($r = 576 \text{ km}$): $v_{\text{orbit}} = \sqrt{63/576} = 0.33 \text{ km/s}$; $v_{\text{hyp}} = \sqrt{4.9^2 + 2 \times 63/576} = 4.92 \text{ km/s} \rightarrow \text{COI} = 4.59 \text{ km/s}$.

Transfer time: $\pi\sqrt{a_t^3/\mu_{\odot}} = 472 \text{ days}$. **Synodic period:** $1/|1/365 - 1/1680| = 466 \text{ days}$ between windows.

13.3 STEP 2 – CHEMICAL OR ELECTRIC?

Chemical budget from LEO: $\$(4.91 + 4.59) \times 1.10 = \$10.45 \text{ km/s}$ (*two propulsion systems, +5% each* : *Physics Handbook : Mission Margins*).

One stage cannot do it. With $s = 0.12$, a stage needs $R < 9.33$, so even hydrolox tops out at $4.415 \ln 9.33 = 9.86 \text{ km/s}$.

Two stages (departure + capture), carrying a ~1,080 kg spacecraft:

Propellant	Mass in LEO	Problem
Hydrolox (RL-10)	26 t	Capture stage boils off 3%/day: $0.97^{472} \approx 0$ left at Ceres. Active Cooling (0.5%/day) still leaves only 9%.
Storable (AJ10)	144 t	No boiloff, but 130× the spacecraft mass; an X-class launch per mission

Solar electric propulsion – Shop T6: 145 mN; 8.3 kg; 4.6 kW; \$8M; Isp 4300s ($v_e = 42.18 \text{ km/s}$); 0.5%† per restart + F:0.35. Licensed [QinetiQ, UK].

13.4 STEP 3 – LOW-THRUST Δv

SEP thrusts continuously, so the Hohmann numbers no longer apply. Use the spiral (Physics Handbook: Low-Thrust Spiral) with $\Delta v = \sqrt{\mu} |1/\sqrt{r_1} - 1/\sqrt{r_2}| = |v_1 - v_2|$.

Launch profile: the launcher puts the spacecraft on an Earth-escape trajectory ($v_\infty \approx 0$), so SEP starts in solar orbit at 1 AU.

Leg	Δv
Heliocentric spiral 1 → 2.77 AU: 29.78 – 17.89	11.89 km/s
Spiral down at Ceres from $v_\infty \approx 0$ to 100 km: v_{orbit}	0.33 km/s
Subtotal	12.22 km/s
With +10% margin (SEP + lander)	13.44 km/s
Lander descent: 0.35×1.10	0.385 km/s

Low thrust costs more Δv than Hohmann (12.2 vs 9.5 km/s). At 4300s it is still cheap in propellant.

13.5 STEP 4 – LANDER

Storable propellant, Isp 320s ($v_e = 3.14 \text{ km/s}$). Engine: **R-4D-15** – 0.5 kN; 5.2 kg; \$150k; 0.1%† + F:1.

Landing gear is 5% of landed mass and structure is 10% of dry mass, so iterate until the dry mass settles at **246 kg**:

$$m_{\text{fixed}} = 200 + 5.2 + 12.3 \text{ (gear)} + 24.6 \text{ (structure)} = 242.1 \text{ kg}$$

$$R = e^{0.385/3.14} = 1.130 \quad m_{\text{wet}} = \frac{242.1 \times 1.130}{1.12 - 0.136} = \mathbf{278.0 \text{ kg}}$$

Propellant 32.1 kg; tanks 3.9 kg.

13.6 STEP 5 – POWER

Size the array so the T6 runs at full power at Ceres.

Load	Power
T6	4,600 W
Avionics (Spacecraft)	200 W
Reaction wheels ×4	400 W
X-band + HGA pointing	41 W
Star trackers ×2 + array tracking	40 W
At 2.77 AU	5.28 kW

Solar flux at Ceres is $1/2.77^2 = 13\%$ of 1 AU, so the array is rated at $5.28 \times 2.77^2 = \mathbf{40.5 \text{ kW at 1 AU}}$.

Shop Solar Panel at 330 W/m²: **122.8 m²** → 430 kg, **\$36.8M**.

13.7 STEP 6 – SPACECRAFT

SEP fixed mass is everything except xenon and its tanks. Bus structure is 10% of the bus dry mass, excluding the lander, which carries its own; iterate as in Step 4.

Item	Shop	Mass (kg)	Cost
Orbiter	given	500	\$10.0M
Lander (wet)	Step 4	278.0	\$5.2M
T6 ×2 (one operating, one spare)	Electric (ion)	16.6	\$16.0M
Solar arrays	Solar Panel	429.8	\$36.8M
Array tracking	Solar Panel Tracking (Large)	10	\$0.1M
Avionics	Avionics (Spacecraft)	50	\$0.5M
X-band + 1 m ² HGA	Communications	20	\$2.0M
Reaction wheels ×4 (need 3)	Reaction Wheel (standard)	120	\$8.0M
Star trackers ×2 (need 1)	Star Tracker	6	\$4.0M
RCS block + 20 kg monoprop	RCS Thruster Block	40	\$1.0M
Lander separation	Separation System	5	\$0.1M
Structure	Spacecraft Structure	141.5	—
SEP fixed mass		1,616.9	

$$R = e^{13.44/42.18} = 1.375 \quad m_{\text{wet}} = \frac{1,616.9 \times 1.375}{1.12 - 0.165} = \mathbf{2,328 \text{ kg}}$$

- Xenon: **635 kg** (\$1.3M at \$2k/kg)
- Xenon tanks: **76 kg**
- **Hardware total: \$85.0M**

Thrust time (constant thrust; mass flow = $0.145/42,180 = 3.44 \times 10^{-6}$ kg/s):

$$t = \frac{635 \text{ kg}}{3.44 \times 10^{-6} \text{ kg/s}} = 1.85 \times 10^8 \text{ s} = \mathbf{2,139 \text{ days}} \approx 5.9 \text{ years}$$

13.8 STEP 7 — LAUNCH AND COST

Launch: rideshare on M-class to Earth escape. Capacity to escape: $22,000 \times e^{-3.2/3.04} = 7,681$ kg (Shop: Payload to Other Destinations).

$$\text{Rideshare} = \frac{\$70\text{M} \times 1.3 \times 2,328}{7,681} = \mathbf{\$27.6\text{M}}$$

Ops (Shop: Operations), from 1 month before launch through 5.9 yr transfer + 1 yr science (≈ 6.98 yr):

- **People:** Cruise $\ell = 0.5 \rightarrow 1$ Ground on ops for 6.98 yr = **\$1.75M**. Science year at Hot $\ell = 3 \rightarrow$ hire 2 more Ground for 1 yr = **\$0.50M**. People total **\$2.25M**.
- **Contact:** Deep-space cruise \$3M/yr $\times 5.98$ yr = **\$17.9M**; science year hot tracking \$8M. Contact total **\$25.9M**.
- **Operations total \$28.2M**.

Build: unit #1 of a custom spacecraft, so hardware $\times 1.2 \times 5.0$ (prototype) = **\$510M**.

	Cost
Spacecraft build	\$510M
Launch	\$28M
Operations	\$28M
Total	$\approx \\$566\text{M}$

Procurement: T6 is Licensed (filing \$1–5M, +6–12 months, DC 90 for a friendly jurisdiction) and N/yr <5 (12–24 months). Start procurement two years before the target window.

Comparison: NASA's Dawn (Vesta + Ceres, launched 2007) cost about \$500M, carried three ion thrusters, and thrusted for about 5.9 years in total.

13.9 STEP 8 — RISK

Full simulation of this spacecraft: **Odds Walkthrough, Examples 1 and 1b**.

- Transfer segment: $F_{\text{seg}} = 2.0/\text{kd}$ over 2.139 kd $\rightarrow E = 4.28$. Expect about four addressable failures; the spare T6, the fourth wheel, and the second tracker exist to absorb them.
- One-shots (launch, array deploy, five T6 starts, separation, lander ignition, gear) sum to 6.8%.

13.10 INSIGHTS

1. **Mass limits chemical, time limits electric.** Chemical needs 26–144 t in LEO; SEP needs 2.3 t but six years of thrusting.

2. **Power is the cost driver.** The solar array is 43% of hardware cost because flux falls as $1/r^2$; at 2.77 AU you need $7.7\times$ the array you would at Earth.
3. **Low-thrust Δv is higher** than Hohmann (12.2 vs 9.5 km/s); Isp 4300s makes it affordable.
4. **Redundancy pays on long missions.** A spare T6 and a fourth reaction wheel turn the most likely failures into degraded operation instead of mission loss.
5. **Operations add up.** ~7 years of cruise people + DSN-class contact (\$28M) still rivals the launch — and that is *without* a fake \$10M/yr LEO-style MOC meter.

13.11 EXTENSIONS

- **Smaller array:** size for 1 AU and throttle at Ceres. Cheaper, but thrust falls with distance; recompute thrust time leg by leg.
- **Two T6 near Earth:** the array has 40 kW at 1 AU; running both thrusters early shortens the transfer.
- **Sample return:** add a Sample Return Capsule (1%†) and an Earth-return leg.
- **Prospecting:** Spectrometer or Sensors payload for a resource survey (Chance: Search).

14. Odds Walkthrough

14.1 Odds Walkthrough: Mission Failure Simulation

Worked examples of Chance mission simulation and Odds drill-down. All rates from the Shop.

Procedure (Chance: Mission simulation)

1. **One-shots** (†) – roll at each activation. Shop % $\rightarrow$ decimal p ; fail if $u < p$. Look up the Odds (†) entry.
2. **Segments** – split the mission into phases where the same parts are in use. Idle parts contribute nothing.
3. **Exposure** – $F_{\text{seg}} = \sum F$; $E = F_{\text{seg}} \times t$ (t in kilodays; 1 kd = 1000 days).
4. **Roll** – u from d100; $X = -\ln(1 - u)$. If $X > E$, the segment is clean.
5. **When / which** – failure at X/F_{seg} into the segment; roll d100 against each line's share F/F_{seg} .
6. **How** – Odds (F) entry for that part. Handle it, update the manifest, and repeat from the failure time.

14.2 Example 1: Ceres transfer

The spacecraft from the Ceres Mission Exercise: one T6 thrusting (a second T6 carried cold), solar arrays, a four-wheel reaction-wheel bank needing 3, two star trackers needing 1. Thrust segment **2,139 days** ($t = 2.139$ kd), Earth departure through Ceres capture.

14.2.1 Step 1 – Manifest

Line	Shop	F (/kd)	Share of F_{seg}
T6	T6	0.35	17.5%
(operating)			
Solar panels	Solar Panel	0.15	7.5%
Avionics	Avionics	0.1	5%
	(Spacecraft)		
X-band	X-band	0.2	10%
Reaction	Reaction	$4 \times 0.2 = 0.8$	40%
wheels, 4	Wheel		
working	(standard)		
	$\times 4$		
Star trackers,	Star Tracker	$2 \times 0.2 = 0.4$	20%
2 working	$\times 2$		
F_{seg}		2.0	

The spare T6 is off and contributes nothing. Structure carries no launch or landing loads during the transfer and is not on the manifest.

$$E = 2.0 \times 2.139 = 4.28 \quad \Pr(\text{clean transfer}) = e^{-4.28} \approx 1.4\%$$

Expect about four addressable failures over nearly six years of thrusting. Most are absorbed by redundancy or cost performance rather than the mission.

14.2.2 Step 2 – First failure

Roll 45 $\rightarrow X = -\ln(0.55) = 0.598 \leq 4.28 \rightarrow$ a failure occurs.

When: $t = X/F_{\text{seg}} = 0.598/(2.0/\text{kd}) = 0.299 \text{ kd} \rightarrow$ **day 299**.

Which – roll 62:

Line	d100 (shares rounded to whole rolls)
T6	00–17
Solar panels	18–24
Avionics	25–29
X-band	30–39
Reaction wheels	40–79
Star trackers	80–99

62 $\rightarrow$ **reaction wheel**.

How – Odds *Reaction wheel failure*, roll 51: Bearing wear 00–44, Motor failure 45–74, Saturation 75–94, Control electronics 95–99 $\rightarrow$ **Motor failure: wheel offline**.

Handle: three wheels working, $K = 3$, so attitude control continues. The bank line drops to $3 \times 0.2 = 0.6$. One more wheel loss puts attitude on the RCS block (Odds: below 3 wheels, use RCS).

14.2.3 Step 3 – Continue from day 299

Remaining: $2.139 - 0.299 = 1.840 \text{ kd}$. New $F_{\text{seg}} = 2.0 - 0.2 = 1.8$. $E = 1.8 \times 1.840 = 3.31$.

Roll 30 $\rightarrow X = -\ln(0.70) = 0.357 \rightarrow 0.357/1.8 = 0.198 \text{ kd} \rightarrow$ **day 497**.

Which – roll 12 (shares now: T6 19.4%, panels 8.3%, avionics 5.6%, X-band 11.1%, wheels 33.3%, trackers 22.2%) $\rightarrow$ **T6**.

How – Odds *SEP thruster failure (ion)*, roll 48: Grid erosion 00–44, Power electronics 45–69, Propellant flow 70–84, Neutralizer 85–99 $\rightarrow$ **Power electronics failure: thruster offline**.

Handle: switch to the spare T6. Starting it is an activation: $0.5\% \dagger \rightarrow$ **roll 37**, $u = 0.37 \geq 0.005 \rightarrow$ lit. The spare is now the operating unit (same $F = 0.35$); the manifest rate stays 1.8. With no spare left, the next power-electronics or neutralizer failure means a degraded or lost transfer.

Continue the same way from day 497 until the segment ends.

Roll	Result
45	Continuous failure at day 299
62	Reaction wheel bank
51	Motor failure: 3 wheels left
30	Next failure at day 497
12	T6
48	Power electronics: switch to spare
37	Spare T6 lights

14.3 Example 1b: Ceres one-shots

Each is rolled at its own moment. Odds ($\nrightarrow$) entries.

Event	p	Odds
M-class launch	0.01	Launch failure (Shop)
Solar array first deploy	0.01	<i>Deployment jam (panels)</i>
T6 starts (initial + 3 coast restarts + capture)	5×0.005	<i>SEP restart failure</i>
Lander separation	0.002	<i>Separation failure</i>
R-4D-15 lander ignition	0.001	<i>Storable start failure</i>
Landing gear	0.02	<i>Landing gear failure</i>
Sum	0.068	

Insurance: launch insurance at $(1.5 \times 0.01 + 0.01) \times (\$28\text{M launch} + \$85\text{M hardware}) \approx \$2.8\text{M}$. Mission insurance for the whole spacecraft is not available: one year of transfer is already $2.0 \times 0.365 = 0.73 > 0.3$. Insuring only a subset of hardware (e.g. both T6 units, $0.35 \times 0.365 = 0.13$ per year) is allowed, at a premium of $1.5 \times 0.13 + 0.01 \approx 0.20$ of their value per year.

14.4 Example 2: Mars lander

Three-day entry and descent. Entry hardware is all one-shot; the engine and avionics are continuous.

One-shots

Item	p	Odds ($\nrightarrow$)
Heat shield	0.005	<i>Heat shield failure</i>
Parachutes	0.01	<i>Parachute failure</i>
Landing gear	0.02	<i>Landing gear failure</i>

Continuous segment

Line	F (/kd)	t (kd)	$F \times t$
AJ10	5	0.001 (one day of burning)	0.005
Avionics (Spacecraft)	0.1	0.003	0.0003
E			0.0053

Roll 02 $\rightarrow X = -\ln(0.98) = 0.020 > 0.0053 \rightarrow$ no continuous failure.

$\Pr(\text{continuous failure}) \approx 0.5\%$ against 3.5% from the one-shots: for short missions, activations carry the risk.

14.5 Example 3: Engine bank on a short burn

Nine Merlin 1D ($F = 120/\text{kd}$ each), need 7 ($m = 3$ engine losses to lose the stage). Burn 180 s, $t = 2.1 \times 10^{-6}$ kd.

First engine-out (bank rate $9 \times 120 = 1080/\text{kd}$): $E = 0.0023 \rightarrow$ about 0.2% per flight. On a hit, roll *Chemical engine failure* (F), set $n = 8$, keep flying.

Stage loss: each engine fails with $p = 1 - e^{-120t} \approx 2.5 \times 10^{-4}$. Three or more of nine: $\binom{9}{3}p^3 \approx 1.3 \times 10^{-9}$.

A bank is not one part with rate 1080/kd: that is the rate of the *first* unit failure, and the stage survives it. Stage loss needs three independent failures inside the same three minutes, which is why engine-out capability makes short burns so safe.

14.6 Example 4: Crewed lunar landing, start to finish

A full mission: several segments, sub-1% rolls, a failure cascade, a degraded engine, and the reputation result.

Locked at commit — objectives: (1) land 2 crew at the target site, (2) 3 days of surface operations, (3) ascend to 100 km lunar orbit for pickup. The company has flown 7 missions (cap: Trusted) and is currently Trusted.

Lander at start of powered descent: 15,000 kg. Descent engine: one **RL-10** (110 kN; 0.25%† + F:60; hydrolox, $v_e = 4.415$ km/s). Ascent engine: one **AJ10** (0.2%† + F:5). Life Support (F:0.5), Avionics (Spacecraft, F:0.1, radiation-hardened), Solar Panel (F:0.15).

Descent budget: 1.7 km/s from 100 km (Physics Handbook: Landing), sized at +10% (two propulsion systems) = 1.87 km/s.

$$m_{\text{prop}} = 15,000 (1 - e^{-1.87/4.415}) = 5,179 \text{ kg} \quad \dot{m} = 110,000/4,415 = 24.9 \text{ kg/s} \quad \text{burn} = 208 \text{ s}$$

14.6.1 Segments

#	Segment	Rolls	Result
1	Launch (H-class)	†Launch 2%: roll 41	Pass
2	TLI ignition	RL-10 0.25%†: roll 00 → refine 31	Pass (see below)
3	Van Allen transit	Electronics radiation: roll 70	Clean (see below)
4	Coast, 3 days	$E = 0.75 \times 0.003 = 0.0023$: roll 55	Clean
5	Powered descent	Ignition: roll 62 pass. Continuous: roll 00 → 01 → 00	Failure (see below)
6	Touchdown	Landing gear 2%†: roll 63	Pass
7	Surface, 3 days	$E = 0.75 \times 0.003$: roll 18	Clean
8	Ascent	AJ10 0.2%†: roll 47 pass. Burn: roll 29	Clean; in orbit

Each segment gets its own roll; a clean segment says nothing about the next.

14.6.2 Segment 2 — a sub-1% one-shot

$p = 0.0025$. Roll **00** puts u in $[0, 0.01)$, which straddles p , so refine: next two digits **31** → $u = 0.0031 \geq 0.0025$ → **pass**. Any first roll of 01–99 passes without refining.

14.6.3 Segment 3 — environmental exposure

Astrodata: Van Allen transit is **2.0 per pass** for electronics; Shop Radiation Hardening multiplies it by 0.3. $E = 0.6$ for the pass, a 45% chance of a radiation hit.

Roll **70** $\rightarrow X = -\ln(0.30) = 1.20 > 0.6 \rightarrow$ clean. A hit would go to Odds *Radiation upset*: a 1-hour reboot in safe mode, harmless on a coast but worth timing away from burns.

14.6.4 Segment 5 — the failure

Exposure: $F_{\text{seg}} = 60 + 0.5 + 0.1 + 0.15 = 60.75/\text{kd}$ over 208 s (2.41×10^{-6} kd) $\rightarrow E = 0.000146$.

Roll: 00 $\rightarrow [0, 0.01]$ straddles $E \rightarrow$ refine **01** $\rightarrow [0.0001, 0.0002]$ still straddles $\rightarrow$ refine **00** $\rightarrow u = 0.000100$. $X = 0.000100 \leq 0.000146 \rightarrow$ **failure**.

When: $X/F_{\text{seg}} = 0.000100/60.75 = 1.65 \times 10^{-6}$ kd $\rightarrow$ **142 s** into the burn.

Which: RL-10 is $60/60.75 = 98.8\%$ of the rate. Roll 47 $\rightarrow$ **RL-10**.

How — the cascade:

Odds entry	Roll	Result
<i>Chemical engine failure</i> — Combustion instability 00–34, Turbopump 35–69, Nozzle 70–91, Injector 92–99	94	<i>Injector failure</i>
<i>Injector failure</i> — Partial blockage 00–74, Coking $\rightarrow$ <i>Combustion instability</i> 75–99	81	<i>Combustion instability</i>
<i>Combustion instability</i> — Oscillation 00–59, Shutdown 60–89, Destructive 90–99	22	Oscillation

Oscillation: thrust -20%, Isp -5%. Throttling to 50% stabilizes it. At full thrust, there is a 20% chance per burn of escalating to shutdown.

A 90–99 on the last roll would have continued: Destructive $\rightarrow$ Propellant feed failure $\rightarrow$ Rupture $\rightarrow$ Propellant tank failure $\rightarrow$ Tank rupture: vehicle loss.**

14.6.5 Handle — recompute, then decide

State at 142 s: 3,544 kg burned; mass 11,456 kg; Δv delivered $= 4.415 \ln(15,000/11,456) = 1.19$ km/s. Still needed: $1.70 - 1.19 = 0.51$ km/s. Propellant left: 1,636 kg (final mass 9,821 kg).

Longer burns cost extra gravity loss: $1.62 \text{ m/s}^2 \times$ added seconds (Physics Handbook: Gravity loss). At nominal thrust the last 0.51 km/s takes 50 s.

Option	Available Δv	Burn for 0.51 km/s	Extra gravity loss	Margin left	Other risk
Full thrust (88 kN, $v_e =$ 4.194)	$4.194 \ln(11,456/9,821) = 0.646$	≈ 63 s	+20 m/s	116 m/s	20%: shutdown at low altitude

Option	Available Δv	Burn for 0.51 km/s	Extra gravity loss	Margin left	Other risk
Throttle	$4.415 \ln(11,456/9,821) = 100$ s		+81 m/s	89 m/s	TWR
50% (55 kN, $v_e = 4.415$)	0.680				$55,000 / (11,456 \times 1.62) = 2.96$

A shutdown on the single descent engine this close to the surface is a crash. Throttling keeps 89 m/s of margin and removes the escalation risk, so the crew **throttles to 50% and lands**.

If the margin under the throttle option had come out negative, the choice would flip: full thrust plus a d100 against 20%, or abort to orbit on the ascent stage.

14.6.6 Outcome

All three objectives were achieved, but the descent engine was damaged on the way down. Gameplay: *hardware damaged but mission completed* → **Partial failure, -1 level**.

	Before	After
Missions flown	7	8
Cap	Trusted	Trusted
Level	Trusted	Established

The next successful mission restores Trusted.

Part III

Reference Materials

15. How Rockets Work

3

How Do Rockets Work?

In Chapter 1, we answered briefly the question: What are rockets? It began by talking about the history of rocket science and engineering and the key incidents in history that led to the development of the modern rocket era. We also discussed some of the types of modern rockets as well as a very basic description of the components and subsystems required to build a rocket. We also talked a little about the types of rocket development programs that are being initiated to assure the future of rocketry.

We then, in Chapter 2, discussed why rockets are needed. We explored the ideas of mission requirements and payloads. We discussed in some detail trajectories and orbits, which are the places and velocities that we would like to place payloads and conduct missions. In order to conduct these missions along these trajectories or from these orbits, we must be able to reach the trajectories and orbits that we desire. That is why rockets are needed.

In this chapter, we will begin to explain how rockets can actually accomplish the goals set forth for them in Chapter 2. How do rockets work? That is a question that has taken centuries of development of the laws of nature and engineering practices to answer. In order to understand the answer to this question, we have to begin our discussion with a dialogue about thrust, momentum, and impulse. From there, we will derive the so-called “rocket equation” and then understand the basics of how rockets work.

3.1 Thrust

First and foremost, thrust is a *force* and is measured in *Newtons*. Older rocket scientists might often revert to using pounds of thrust, but we will stick to the International System of Units (SIs) here. It is a force generated by some propulsive element in order to overcome other forces acting on a body in order to manipulate that body’s position and velocity vector. It is the force that is used to propel a rocket or a spacecraft to the destination trajectory or orbit or landing site desired. Airplanes use propellers or jet engines to generate thrust. Rockets use rocket engines to generate thrust.

Mathematically speaking, thrust is the net external force acting on an object that can be calculated as the rate of change of the momentum of the body. In other words,

$$p = mv \quad (3.1)$$

where p is the momentum of a mass, m , moving with scalar velocity, v . The average force on the mass, therefore, is

$$\text{Thrust} = F = ma = \frac{dp}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt}. \quad (3.2)$$

Equation 3.2 describes the conservation of momentum of a mass that is moving with a varying velocity plus the possibility that some of the system is at constant velocity, but changing in mass. What does this really mean?

Figure 3.1 shows a rocket with both fuel and oxidizer tanks. As the fuel is burned with the oxidizer in the combustion chamber, high-pressure gases are created from the chemical combustion process. As the gases push against the combustion chamber walls with higher pressure than the outside ambient pressure, they are forced out the opening at the bottom of the chamber called the *throat*, through a nozzle where the velocity of the exhaust is accelerated, and then out into the space behind the rocket. It is the escaping of these

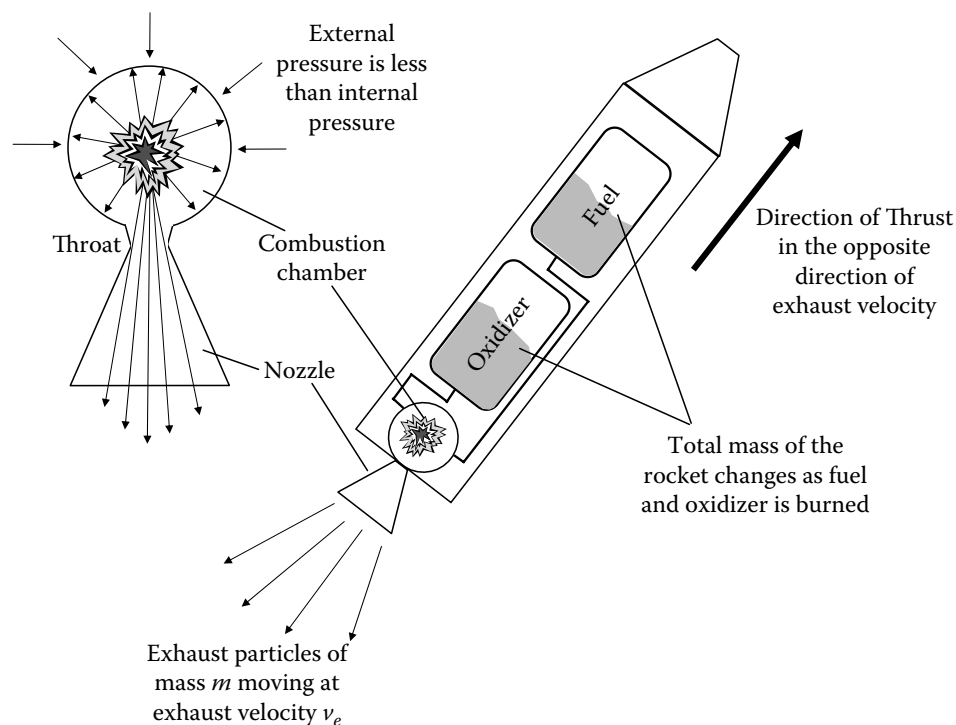


FIGURE 3.1

The liquid rocket engine's system mass varies with time as the fuel and oxidizer are burned in the combustion chamber generating exhaust gases, which are accelerated through the nozzle generating thrust.

highly accelerated exhaust gases that propel the rocket through Newton's Third Law.

It should be noted here that Newton's Third Law is a restatement of the law of conservation of momentum for this system. The simplest rocket would have one particle for propellant (like the baseball and boat example given in Chapter 2). As the propellant mass is thrown from the main mass of the rocket, the rocket is accelerated in the other direction and momentum is conserved. As the calculation of momentum conservation and velocity change for a ball being thrown from a boat was already discussed in Chapter 2, we will not redo it here. If the concept is confusing to the reader at this point, then it can be reexamined.

With Figure 3.1 in mind, we will reconsider Equation 3.2. The thrust equation can be rewritten as

$$F_{Thrust} = \frac{dm}{dt} v + m \frac{dv}{dt} = \dot{m} v_e + m \dot{v}_e. \quad (3.3)$$

For convenience, the rate of change convention of placing a dot over the variable is used, and for example, the "m-dot" is used to describe the time rate of change of mass from the rocket. The escaping gases have an exhaust velocity v_e . The left part of the right-hand side of Equation 3.3 is a useful and understandable quantity. The m-dot tells us the rate at which the rocket engine is burning fuel and oxidizer and is controlled by the throttle of the vehicle. This is the same type of throttling that occurs when you step on the accelerator pedal of an automobile. When you hear the expression: "Roger, Shuttle, go at throttle-up"—this is exactly what is taking place. The astronauts onboard the Space Shuttle are increasing the throttle or, in other words, they are increasing the m-dot. The exhaust velocity is a constant and is determined by the chemicals used in the combustion process as well as the geometrical design of the chamber, throat, and nozzle. Once the rocket engine is designed and built, the exhaust velocity does not change unless a different fuel or oxidizer is used (which is typically not good for the rocket as they are not usually designed but for one fuel/oxidizer combination).

The right part of the right-hand side of Equation 3.3 is a bit more erroneous and does not immediately suggest to us design parameters. However, "v-dot" is acceleration by definition and, thus, the right part can be described as the force component due to the exhaust mass escaping the rocket that is being accelerated by the diverging nozzle. Another way to look at this is that the exhaust gas is quickly flowing out of this nozzle and is being accelerated faster and faster as it approaches the exit. When the gas exits the nozzle the flowing gas leaving the rocket in the opposite direction then pushes the rocket forward. To quantify this, we must first realize that force can also be written as pressure, P , multiplied by the area, A , that the pressure is incident upon or

$$F = PA. \quad (3.4)$$

Therefore, if the pressure inside the nozzle just before it exits is the exhaust pressure P_e , and the pressure outside the nozzle is P_o , then the force due to the pressure difference on the nozzle exit surface area A_e is

$$F_{\text{nozzle}} = (P_e - P_o) A_e. \quad (3.5)$$

Substituting the right-hand side of Equation 3.5 into Equation 3.2 gives us the rocket thrust equation

$$F_{\text{Thrust}} = \dot{m}v_e + (P_e - P_o) A_e. \quad (3.6)$$

Equation 3.6 is an important design equation for rocketry. From it we can design a rocket engine in general terms. The exhaust velocity and pressure is determined by the fuel/oxidizer combination, throat, and nozzle design. The nozzle design drives the definition of the exit area of the nozzle. Therefore, from Equation 3.6, a rocket can be designed to generate a particular desired thrust. The equation is also powerful in allowing us to analyze a particular engine and determine the thrust that it can generate given its particular design.

3.2 Specific Impulse

As we discussed above, thrust is the force generated by the rocket engine that propels a rocket along its trajectory through the air and space. Specifically, the engine houses a combustion process where a gas is heated and expanded and then forced out the rear of the rocket in the opposite direction as that of the motion of the rocket itself. As we will discuss in Chapter 4, this gas is forced out the back of the rocket through a converging, diverging nozzle system that accelerates the gas flow. For now, just consider the fact that the gas flow is accelerated out of the rocket engine and that that is the main purpose of the rocket engine—to accelerate exhaust gas flow.

To better understand the thrust generated by this accelerated gas flow, a few physics concepts need to be discussed. In classical physics, there is a phenomenon where momentum is imparted to a baseball by a bat (assuming the batter keeps his eye on the ball) called *impulse*. Impulse, I (sometimes called *total impulse*), in its purist sense is defined as the total integrated force with respect to time and is written as

$$I = \int F dt. \quad (3.7)$$

Here, F is force as a constant or a function of time and dt is the incremental time change variable. Recall from Newton's Second Law that force can be written as the derivative of momentum with respect to time or

$$I = \int F dt = \int \frac{dp}{dt} dt = \int dp = \Delta p. \quad (3.8)$$

Equation 3.8 is referred to as the *impulse-momentum theorem*. And once again, the theorem basically tells us that a force applied to an object over a given amount of time produces an effect and that effect is an *impulse*. Another way to think of impulse is that it is the change in momentum of an object due to an applied force. Recall the baseball's momentum being changed by hitting a bat. Also note from Equation 3.8 that impulse will have the same units as momentum of kg m/sec or a N·s. Integrating Equation 3.8 yields

$$I = Ft = \Delta p. \quad (3.9)$$

Considering a force generated by a changing mass with a constant velocity, the impulse can be written as

$$I = \int_{m_f}^{m_i} v \frac{dm}{dt} dt = \Delta p = (m_i - m_f) v. \quad (3.10)$$

As our discussion continues, it will become clear why Equation 3.10 might be useful. Clearly, if we have a rocket that is in need of a course change or even a lift-off, we must change its momentum and Equation 3.10 shows us how. If we start off with an initial mass for the rocket and apply an impulse by ejecting propellant mass out of the back of it at a constant exhaust velocity, we can rewrite the equation as

$$I = (m_i - m_f) v = \Delta m_{\text{propellant}} v_e. \quad (3.11)$$

Solving for the exhaust velocity results in

$$\frac{I}{\Delta m_{\text{propellant}}} = v_e. \quad (3.12)$$

Before we go farther with this line of reasoning, we need to identify something. We said in Equation 3.6 that we defined thrust as something more

than simply the $\dot{m}$ times the exhaust velocity. Reexamining that equation and defining a new parameter called the *equivalent velocity* or sometimes the *effective exhaust velocity*, C , we rewrite Equation 3.6 as

$$F_{Thrust} = \dot{m}v_e + (P_e - P_o)A_e = \dot{m}C \quad (3.13)$$

and we see that Equation 3.11 and Equation 3.12 should really be the *equivalent velocity* rather than just the exhaust velocity, thus,

$$\frac{I}{\Delta m_{propellant}} = C. \quad (3.14)$$

This equation tells us that the total impulse imparted to a rocket divided by the propellant mass ejected is equal to the equivalent velocity. The relation is useful in describing the total rocket thrust, but it doesn't really tell us anything about the rocket itself. By defining a new parameter, we can make some very powerful assessments with Equation 3.14.

That parameter is known as the specific impulse, I_{sp} . This is a more useful parameter (given in seconds only) and is written as

$$I_{sp} = \frac{I}{\Delta m_{propellant}g} = \frac{C}{g}. \quad (3.15)$$

What Equation 3.15 tells us is that the I_{sp} of a rocket engine is the total number of seconds that the rocket can deliver thrust equal to the weight of the total propellant mass under acceleration due to one standard Earth gravity, g . This is an efficiency number that we use to describe rocket engines. The higher the I_{sp} the more efficiently the engine can apply Δv to the spacecraft. Table 3.1 shows some specific impulse values for various rocket systems. From that data it becomes clear that specific impulse is not the only important parameter when it comes to discussing rockets. For example, a launch vehicle engine will have I_{sp} typically around 200 to 500 seconds. An ion engine for a deep space mission will have much higher I_{sp} values, upwards of 3,000 seconds. Why are they so different and why would we use one over the other?

Launch vehicles typically are employed to lift very heavy payloads into orbit or on an interplanetary trajectory as quickly as possible. The key component needed for such missions is thrust—as much thrust as usually can be applied within engineering capabilities. Applying this much thrust in such

TABLE 3.1

Specific Impulse for Various Rockets

Rocket	I_{sp} in sec
SSME	363
RS-68	365
SRB	269
NSTAR	3,100
NERVA	800

a short time requires a large amount of propellant mass. This is why launch vehicle rockets are mostly propellant and oxidizer with small areas on top for payloads. An example of this is the Delta IV heavy launch vehicle that employs three common booster core liquid hydrogen fuel-, liquid oxygen oxidizer-driven Boeing/Rocketdyne RS-68 engines as discussed in Chapter 1. Each of the three engines can supply just under 3.4 mega-Newtons (MN) of thrust with an I_{sp} of 362 sec at sea level.

On the other hand, interplanetary missions (after launch) typically need to apply thrust continuously for a long period of time. These are usually the small payloads that are atop the launch vehicles, thus they have very little mass budget for fuel. This means that they cannot apply large thrusts for long periods of time or they will run out of fuel. Hence, a more propellant efficient engine, such as an ion thruster, is needed. The ion thrusters use small amounts of propellant mass at a time, but accelerate that mass to very high equivalent velocities. Figure 3.2 illustrates images of NASA's Deep Space Probe 1 that used an ion engine that only generated 0.09 N of thrust, but had an I_{sp} of over 3,100 sec.

3.2.1 Example 3.1: I_{sp} of the Space Shuttle Main Engines

The three Space Shuttle Main Engines (SSMEs) of the Orbiter each provide about 1.8 MN of thrust with an I_{sp} of 363 sec at sea level. What is the mass flow rate of an SSME?

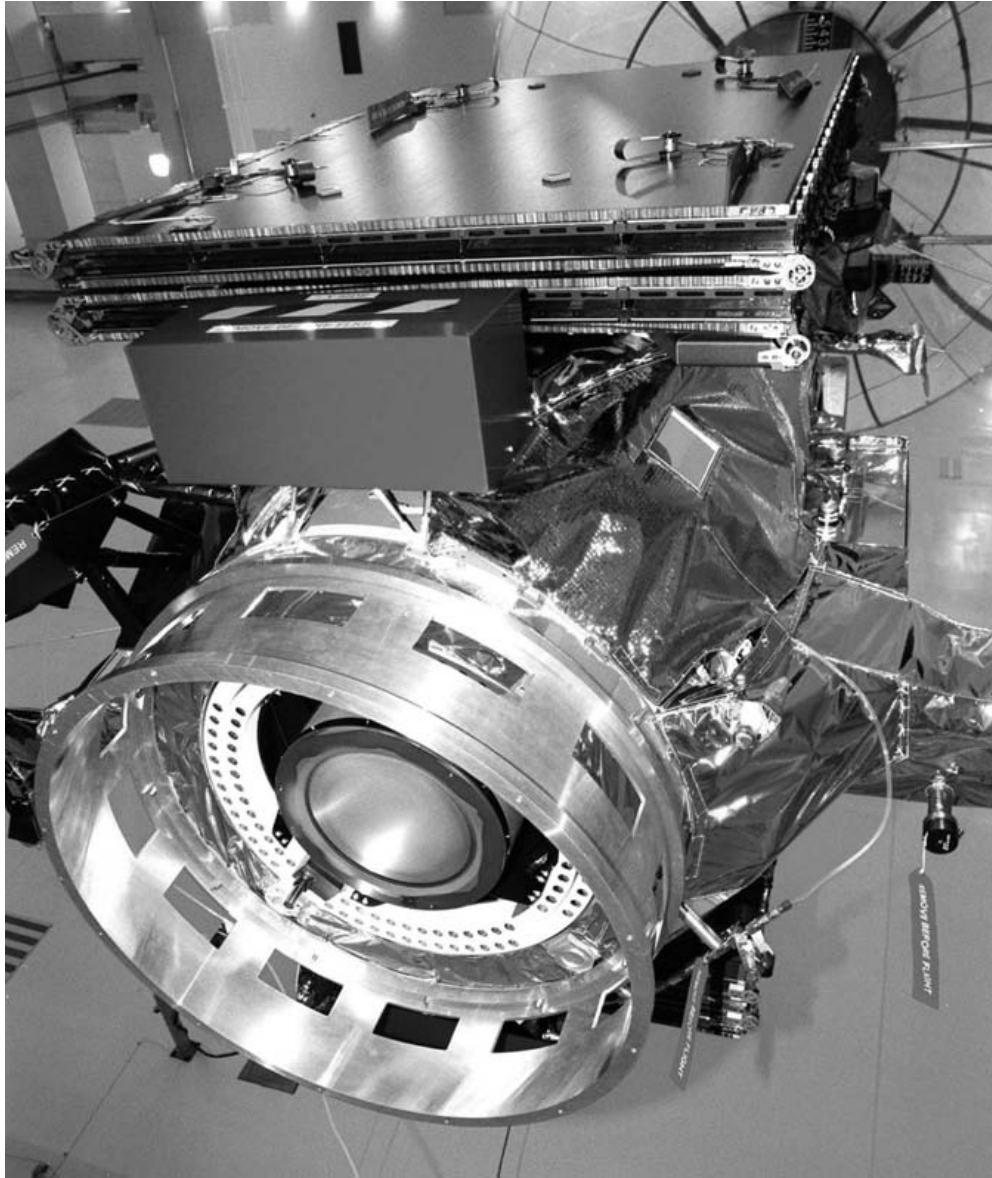
The first step is to determine the equivalent velocity, C , of the engine. From Equation 3.15, we see that $C = g I_{sp} = 3557.4$ m/sec. Then from Equation 3.13, we can solve for the mass flow rate, $\dot{m}$, which is

$$\dot{m} = \frac{F_{thrust}}{C} = \frac{1.8 \times 10^6}{3557.4} = 505.99 \text{ kg/sec.}$$

In other words, one SSME uses about a half ton of propellant each second. Don't forget that there are three of them.

3.3 Weight Flow Rate

What we have learned from Sections 3.1 and 3.2 is that the two parameters, thrust and specific impulse, are key in defining rocket engines for particular applications. Knowing these parameters tells us a lot on how to size the rocket engine for a particular mission. There is another parameter that we should also mention here. Let's reconsider Equation 3.15 by rewriting it as

**FIGURE 3.2**

The Deep Space Probe 1 before it was launched in 1998. This view gives a good vantage point of the ion engine that fired successfully for 678 days. (Photo courtesy of NASA.)

$$I_{sp} = \frac{I}{\Delta m_{\text{propellant}} g} = \frac{C}{g} = \frac{F_{\text{thrust}} \Delta t}{\Delta m_{\text{propellant}} g} = \frac{F_{\text{thrust}}}{\frac{\Delta m_{\text{propellant}}}{\Delta t} g}. \quad (3.16)$$

Realizing that the change in propellant mass over the period of time thrusting occurs multiplied by the gravitational acceleration of one standard Earth gravity is the parameter known as *weight flow rate*, $\dot{W}$, then it becomes

$$I_{sp} = \frac{F_{thrust}}{\dot{W}}. \quad (3.17)$$

Another way to look at Equation 3.17 is that the *weight flow rate* is the ratio of rocket thrust to specific impulse,

$$\dot{W} = \frac{F_{thrust}}{I_{sp}}. \quad (3.18)$$

Two different rocket engines will likely have different values for I_{sp} , thrust, and weight flow rate, but it is these three parameters that enable rocket engineers to begin initial sizing of the engine. The thermodynamic properties of the engine (combustion, gas dynamics, and nozzle design) will determine the I_{sp} . The overall weight of the spacecraft and rocket combination will drive the thrust requirement. And, having these two values allows us, through Equation 3.18, to determine the weight flow rate of propellant mass needed, which will lead to design knowledge about how big the nozzle throat of the rocket must be. Figure 3.3 shows a graphic of the thrust versus the specific impulse for a few engines. From Equation 3.18, we see that the weight flow rate is the slope of this graph. This is a useful design tool telling us that, for a given thrust and specific impulse, we will need to construct an engine that can handle the flow rates shown on the graph and determined by Equation 3.18.

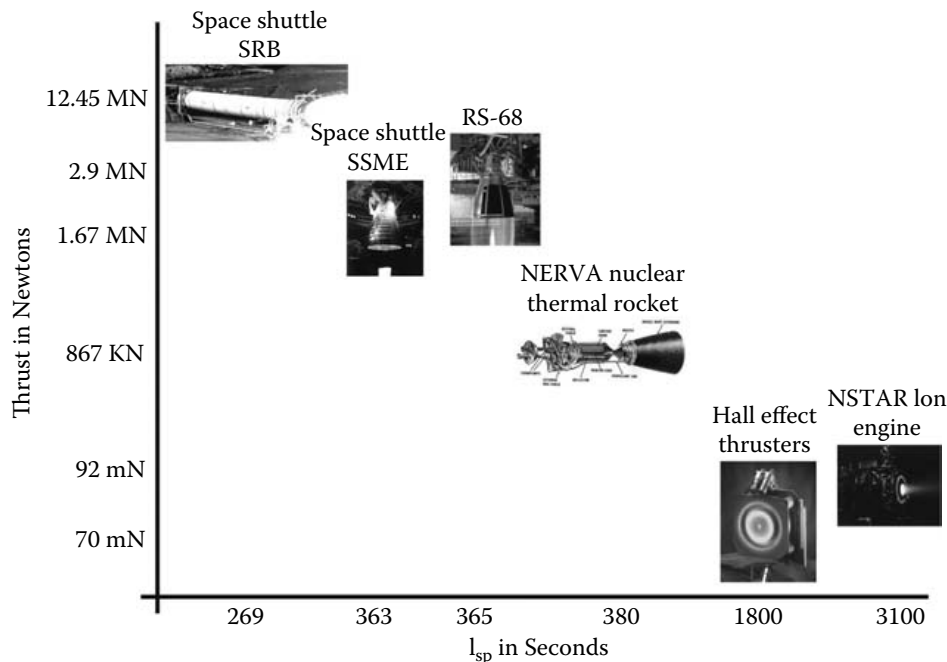


FIGURE 3.3

Various rocket engines shown as thrust versus I_{sp} . (Images courtesy of NASA.)

3.4 Tsiolkovsky's Rocket Equation

As discussed in Chapter 1, the father of rocket science was the Russian mathematics teacher Konstantin Eduardovich Tsiolkovsky. In 1903, he published a paper where he derived the so-called rocket equation. It is this derivation that shows us the basis for rocket propulsion. He even went so far as to describe using multistaged rockets, which we will discuss in the next section.

It turns out that the derivation for the rocket equation is not an extremely complicated one. Most certainly, the tools were available to make the discovery as far back as Newton's time. In fact, that is where the derivation starts, with Newton's Second Law of Motion.

Consider the rocket system shown in Figure 3.4 where there is a rocket vehicle and propellant with total mass, M , an equivalent exhaust velocity, C , and a mass flow rate, $\dot{m}$. The thrust then follows Newton's Second Law and is written as

$$F_{Thrust} = \dot{m}C = \frac{dM}{dt}C. \quad (3.19)$$

Also applying Newton's Second Law in that the total force of the rocket can be defined by the total mass of the rocket vehicle and propellant times the total acceleration, then

$$F = Ma = M \frac{dv}{dt} \quad (3.20)$$

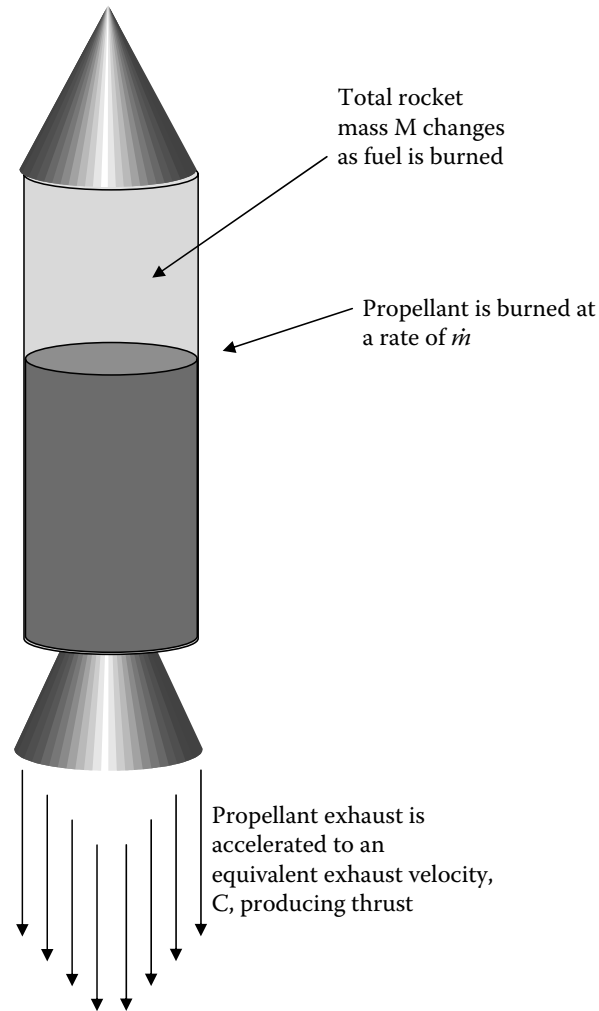
where v is the velocity of the rocket. Setting the right-hand sides of Equation 3.19 and Equation 3.20 equal to each other and realizing that they are forces in the opposite direction (Newton's Third Law) gives

$$M \frac{dv}{dt} = -\frac{dM}{dt}C. \quad (3.21)$$

Simplifying the above equations leads to the following differential equation

$$dv = -C \frac{dM}{M}. \quad (3.22)$$

Assuming that the rocket starts out with a velocity of v_o and ends with velocity v_f and the initial mass is M_o and final mass is M_f , then we can solve Equation 3.22 by integrating it through these limits:

**FIGURE 3.4**

As the rocket engine fires, the total mass of the rocket decreases, propellant is burned, and the exhaust is accelerated.

$$\int_{v_o}^{v_f} dv = -C \int_{M_o}^{M_f} \frac{dM}{M}. \quad (3.23)$$

Integrating and applying the limits results in the rocket equation

$$v_f - v_o = -C \left(\ln(M_f) - \ln(M_o) \right) = C \ln \left(\frac{M_o}{M_f} \right). \quad (3.24)$$

Realizing that the left-hand side of Equation 3.24 is the change in velocity, Δv , then it can be rewritten as

$$\Delta v = C \ln \left(\frac{M_o}{M_f} \right). \quad (3.25)$$

In many textbooks or when talking to rocket scientists and engineers, it is likely to hear the ratio of the initial mass of the rocket to the final mass in the argument of the natural logarithm of Equation 3.25 as the *mass ratio*, or sometimes the *propellant mass ratio*, and sometimes as the *mass fraction*. In some cases, the mass ratio is given as MR , making Equation 3.25 look like

$$\Delta v = C \ln(MR). \quad (3.26)$$

For the purposes of clarity in this book, we will stick to Equation 3.25.

Now, simplifying Equation 3.25 by dividing both sides by C and raising each side to the exponent yields

$$M_o = M_f e^{\frac{\Delta v}{C}}. \quad (3.27)$$

From Equation 3.15, we see that $C = g \cdot I_{sp}$. Substituting this value into Equation 3.27 gives

$$M_o = M_f e^{\frac{\Delta v}{g I_{sp}}}. \quad (3.28)$$

This is yet another formulation of the rocket equation and is expressed in useful rocket science and engineering terms. As we saw in Chapter 2, the Δv is an all-important parameter and we are realizing within this chapter the importance of the other terms in Equation 3.28.

Now, let's reconsider this derivation taking into account the gravitational force on a launching rocket. We will start by writing the total force on the rocket as

$$F = Ma = M \frac{dv}{dt} = -F_{thrust} - Mg. \quad (3.29)$$

Again, force due to thrust is the right-hand side of Equation 3.19, thus Equation 3.29 becomes

$$M \frac{dv}{dt} = -C \frac{dM}{dt} - Mg. \quad (3.30)$$

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Simplifying and integrating

$$\int_{v_o}^{v_f} dv = -C \int_{M_o}^{M_f} \frac{dM}{M} - g \int_{t_o}^{t_f} dt, \quad (3.31)$$

$$(v_f - v_o) = C \ln \left(\frac{M_o}{M_f} \right) - g(t_f - t_o). \quad (3.32)$$

Again realizing the left-hand argument is Δv and the time argument on the right-hand side is the Δt or time of rocket burn, t_b , then Equation 3.32 becomes

$$\Delta v = C \ln \left(\frac{M_o}{M_f} \right) - g t_b. \quad (3.33)$$

Finally, moving the time of burn segment to the right-hand side, taking the exponent of each side, and substituting in for C results in the adjusted for gravity rocket equation

$$M_o = M_f e^{\frac{\Delta v + g t_b}{g I_{sp}}}. \quad (3.34)$$

The initial and final mass notation in Equation 3.28 and Equation 3.34 might become confusing especially because the mass diminishes with time. It is common to see these masses described as the “full-of-fuel” or just “full” mass of the rocket and the “empty” mass of the rocket. Thus, the equations are sometimes written as

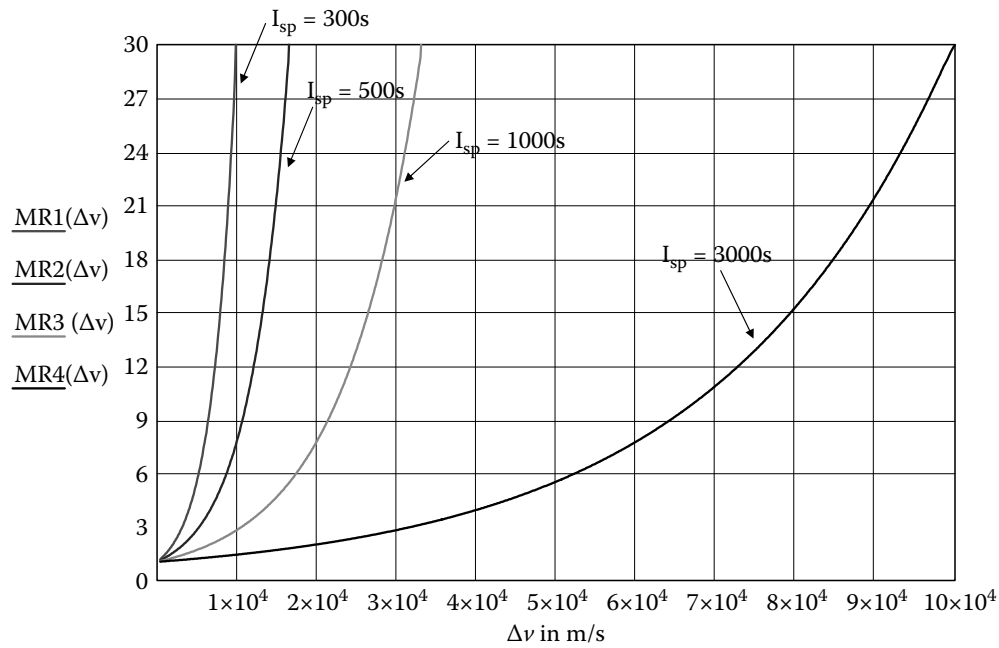
$$M_{full} = M_{empty} e^{\frac{\Delta v}{g I_{sp}}} \quad (3.35)$$

and

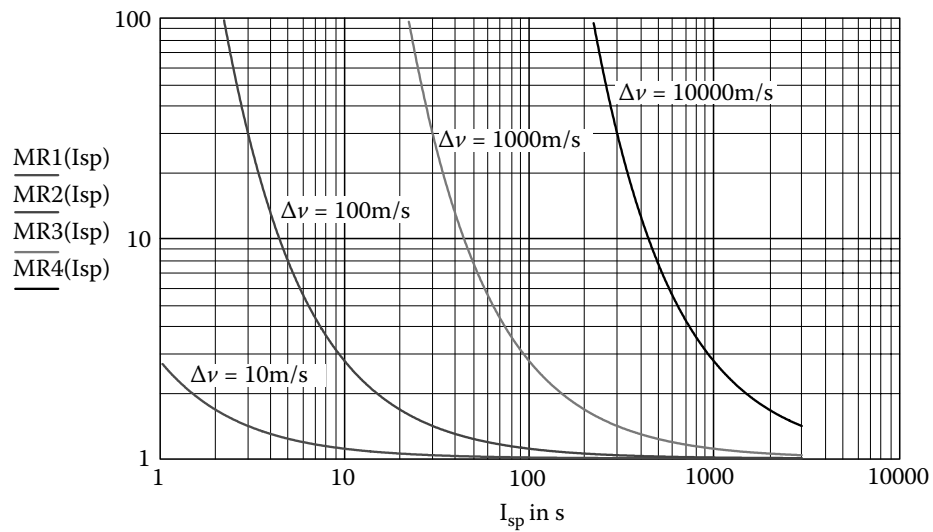
$$M_{full} = M_{empty} e^{\frac{\Delta v + g t_b}{g I_{sp}}}. \quad (3.36)$$

Figure 3.5 and Figure 3.6 show graphs of the mass ratio as a function of the Δv and the specific impulse, respectively. It is clear that the Δv goes up as the MR goes up, but the I_{sp} goes down as MR goes down.

Equation 3.36 is a fairly useful tool in that it can be used to design parameters around a mission. Assuming that we use the procedures found in Chapter 2 to develop the Δv needed for a space mission for a particular craft,

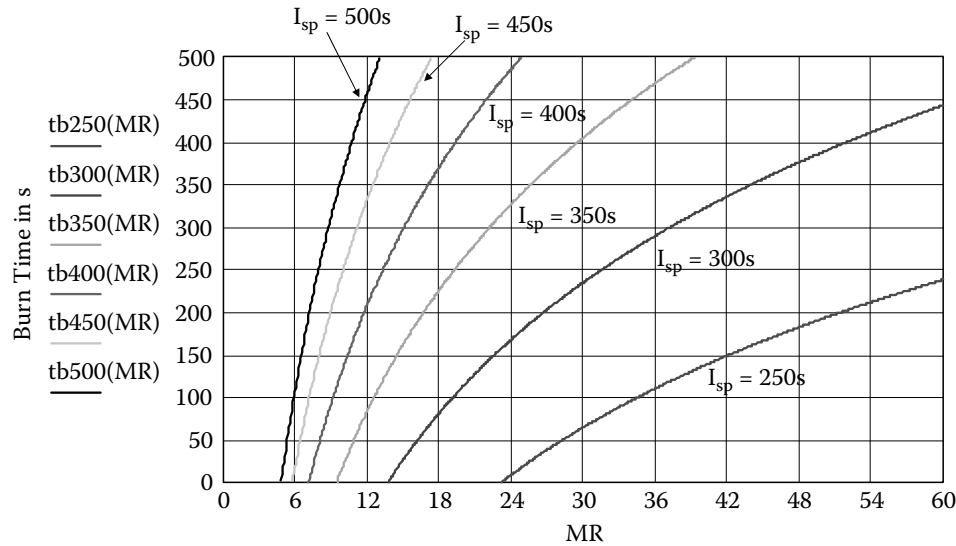
**FIGURE 3.5**

Mass ratio, MR , versus Δv for various I_{sp} values.

**FIGURE 3.6**

Mass ratio, MR , versus I_{sp} for various Δv values.

then we can use Equation 3.36 to determine the MR required to do this, at what I_{sp} , and for how long of a burn time. Taking the natural log of both sides gives us the burn time equation

**FIGURE 3.7**

Burn time in seconds versus the mass ratio.

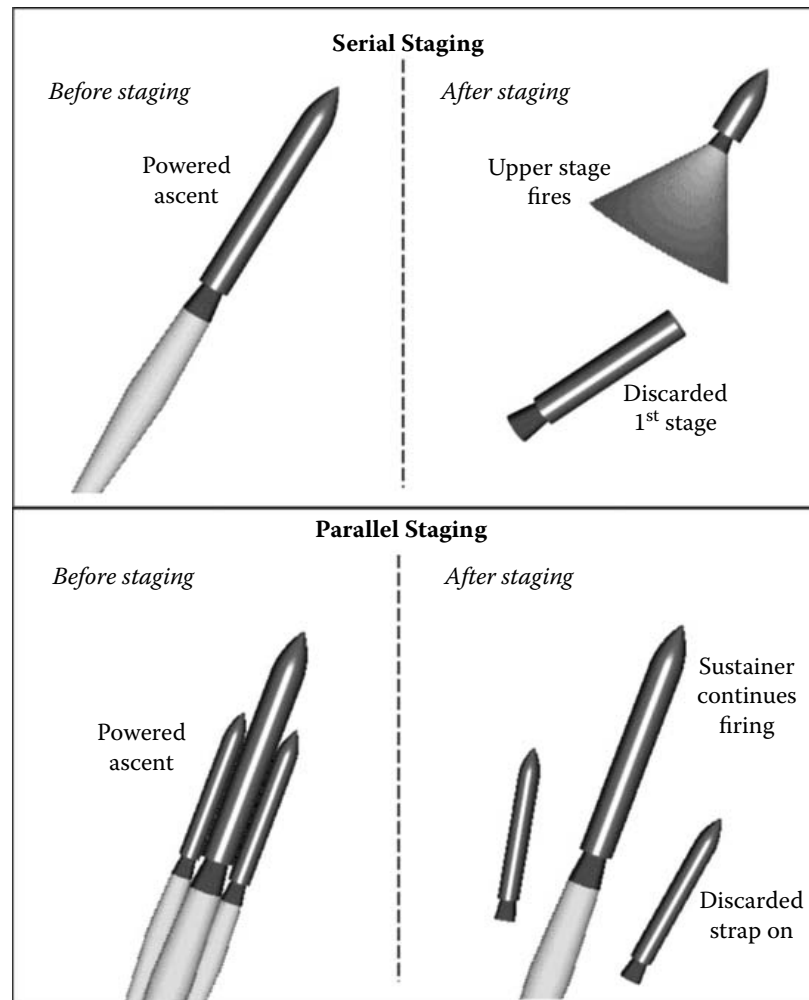
$$t_b = \ln \left(\frac{M_{full}}{M_{empty}} \right) I_{sp} - \frac{\Delta v}{g}. \quad (3.37)$$

Figure 3.7 shows a graph of the burn time as a function of the mass ratio for a required Δv of 7,700 m/sec and for several values of I_{sp} . Clearly, as the specific impulse increases, the burn time and mass ratio reduces.

3.5 Staging

We now see that as a launch vehicle rocket lifts itself up it discards a lot of fuel and oxidizer mass as can be seen by the MR of the system. From some of the figures of launch vehicle rockets shown in Chapter 1, it is quite clear that these systems are very large in size and, even though they are made of lightweight space-age materials, they are still quite heavy. Therefore, it makes sense that throwing away the empty fuel and oxidizer tanks, and the structure supporting them, and the engines that used them would enable the use of a secondary rocket system that has a smaller initial or full mass and, therefore, changing the performance of the rocket as described by Equation 3.35 to Equation 3.37.

This concept is called *staging* of the launch vehicle. Figure 3.8 illustrates the two basic types of staging that are typical with rocket systems. The first type of staging is *serial staging* and the second is *parallel staging*. Serial

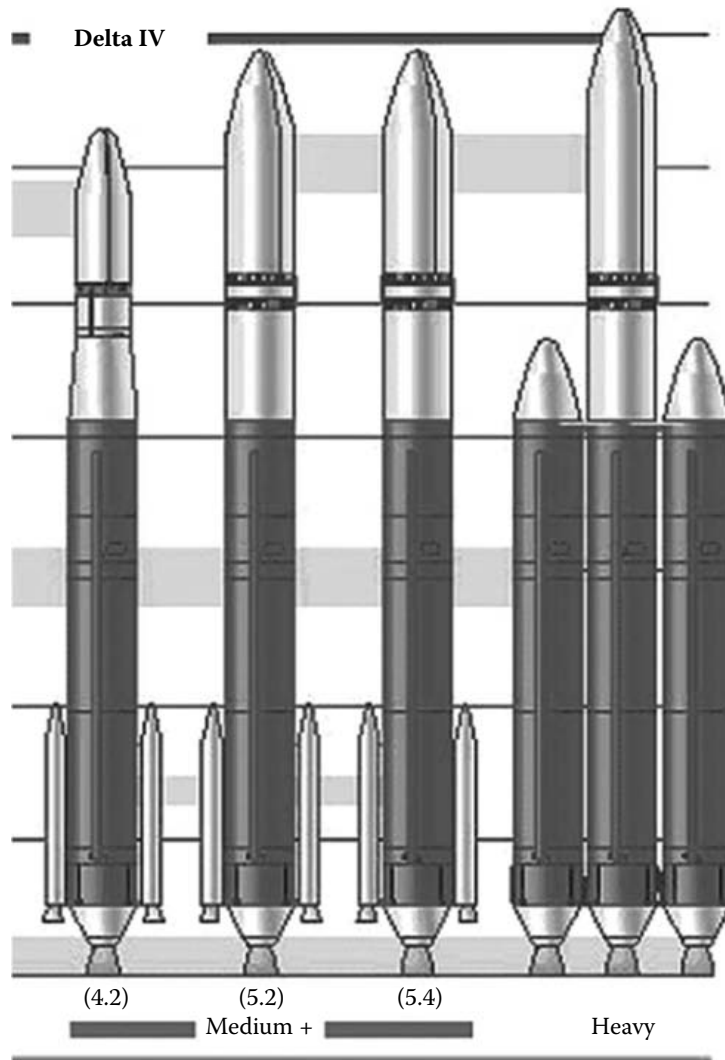
**FIGURE 3.8**

Two standard types of booster staging. (Photo courtesy of NASA.)

rocket staging is a system that stacks stages one atop the other, while parallel staging is a system that straps boosters beside each other. The Space Shuttle is an example of parallel staging, while the Saturn V of the Apollo program was serial staged rockets.

There is another type of staging, which we will call *hybrid staging*. Hybrid staging is a combination of the two basic types. An example of a hybrid staged rocket system is the Delta IV family of launch vehicles as discussed in Chapter 1 and shown in detail in Figure 3.9. This vehicle has parallel strap-on boosters (solids Gem 60s for the normal and medium class and liquid RS-68s for the heavy class) as well as an upper stage liquid booster (RL-10B-2 engine).

Before we can develop a model for staging rockets, we need to discuss the impact staging has on some of the rocket's parameters. Specifically, we should take a closer look at describing the rocket's mass. The basic components of

**FIGURE 3.9**

The Delta IV medium + to heavy rockets use hybrid staging including parallel strap-on boosters and serial upper stage boosters. (Photo courtesy of NASA.)

the rocket vehicle are the structure, which houses the engines and tanks for each stage; the fuel and oxidizer, which we will just call propellant for now; and the payload, which consists of instruments and, sometimes, astronauts. Each of these contribute to the rocket system's mass and, thus, the total full mass of the rocket system can be written as

$$M_{total} = M_{structure} + M_{propellant} + M_{payload}. \quad (3.38)$$

For a two-staged rocket, Equation 3.38 becomes

$$M_{total} = M_{stage1structure} + M_{stage1propellant} + M_{stage2structure} + M_{stage2propellant} + M_{payload}. \quad (3.39)$$

For convenience from this point, we will use subscripts of s for structure and p for propellant with a number to denote the stage it represents. Hence, Equation 3.39 is more simply written

$$M_{total} = M_{s1} + M_{p1} + M_{s2} + M_{p2} + M_{payload}. \quad (3.40)$$

For a multistaged rocket, Equation 3.40 can be written as

$$M_{total} = M_{s1} + M_{p1} + M_{s2} + M_{p2} + \dots + M_{sN} + M_{pN} + M_{payload}. \quad (3.41)$$

Now consider how a system like the one described by Equation 3.41 impacts the calculation of the mass ratio and, therefore, the rocket equation. The approach to account for this is to develop a mass ratio calculation for each stage. The MR for the initial state of the multistaged rocket for the first stage burn is

$$\begin{aligned} MR_1 &= \frac{M_{total}}{M_{total} - M_{p1}} \\ &= \frac{M_{s1} + M_{p1} + M_{s2} + M_{p2} + \dots + M_{sN} + M_{pN} + M_{payload}}{M_{s1} + M_{s2} + M_{p2} + \dots + M_{sN} + M_{pN} + M_{payload}}. \end{aligned} \quad (3.42)$$

After the first stage is expended and jettisoned, the rocket then becomes smaller and the new mass ratio becomes

$$MR_2 = \frac{M_{s2} + M_{p2} + \dots + M_{sN} + M_{pN} + M_{payload}}{M_{s2} + \dots + M_{sN} + M_{pN} + M_{payload}}. \quad (3.43)$$

Likewise, the mass ratio for the N^{th} stage is

$$MR_N = \frac{M_{sN} + M_{pN} + M_{payload}}{M_{sN} + M_{payload}}. \quad (3.44)$$

Using the above mass ratio formulas for the multiple stages along with Equation 3.36, the performance of the rocket at each stage can be determined. This approach allows rocket scientists and engineers to determine how many stages are needed to achieve the final desired Δv for the mission payload. We should note that there is an optimum mass ratio for each of the stages. It is outside the scope of this text to prove this here, but it turns out that the optimum for the rocket system is to design it such that the mass ratios for each stage are equal to one another if each stage uses the same type of propellant or engines with similar performance parameters. Using this knowledge,

rocket engineers can determine what mass for structure and propellant can be allowed for each stage.

3.5.1 Example 3.2: The Two-Stage Rocket

Given a two-stage rocket with each stage of equal mass and same engines, find the equation that describes the total Δv achieved by the rocket after second stage burnout.

Start with Equation 3.35 and solve for the total Δv , which is

$$\Delta v_{total} = gI_{sp} \ln \left(\frac{M_{full}}{M_{empty}} \right). \quad (3.45)$$

The mass ratio for stage 1 is

$$MR_1 = \frac{M_{total}}{M_{total} - M_{p1}} = \frac{M_{s1} + M_{p1} + M_{s2} + M_{p2} + M_{payload}}{M_{s1} + M_{s2} + M_{p2} + M_{payload}}. \quad (3.46)$$

So, the Δv for the first stage is

$$\Delta v_1 = gI_{sp} \ln \left(\frac{M_{s1} + M_{p1} + M_{s2} + M_{p2} + M_{payload}}{M_{s1} + M_{s2} + M_{p2} + M_{payload}} \right). \quad (3.47)$$

The mass ratio for stage 2 is

$$MR_2 = \frac{M_{s2} + M_{p2} + M_{payload}}{M_{s2} + M_{payload}}. \quad (3.48)$$

The Δv for stage 2 is then

$$\Delta v_2 = gI_{sp} \ln \left(\frac{M_{s2} + M_{p2} + M_{payload}}{M_{s2} + M_{payload}} \right). \quad (3.49)$$

The total Δv for the two-stage rocket, therefore, is written as

$$\begin{aligned} \Delta v_{total} &= \Delta v_1 + \Delta v_2 \\ &= gI_{sp} \ln \left(\frac{M_{s1} + M_{p1} + M_{s2} + M_{p2} + M_{payload}}{M_{s1} + M_{s2} + M_{p2} + M_{payload}} \right) \\ &\quad + gI_{sp} \ln \left(\frac{M_{s2} + M_{p2} + M_{payload}}{M_{s2} + M_{payload}} \right). \end{aligned} \quad (3.50)$$

3.6 Rocket Dynamics, Guidance, and Control

To this point in our development of rocket science and engineering understanding, we have discussed details of the anatomy of a rocket (see Section 1.3, Chapter 1) and, in this chapter, we have discussed the basic components of propulsion and staging. At the most basic level, a rocket can be described as four major subsystems:

1. Payload system that carries instruments and astronauts
2. Propulsion system that contains the engines, pumps, and propellants
3. Structural system that houses all components and is also called the “frame”
4. The guidance and control system

All of the components interact with each other and are integral subsystems to the overall rocket vehicle system.

The guidance and control system is impacted by the rocket design because tanks, structure, propellant mass, engine mass, and the location within or on the rocket of all these units change the rocket vehicle system’s center of gravity. Figure 3.10 shows a graphic of a rocket with all the dynamic forces impinging on it. These forces include aerodynamic lift, aerodynamic drag, the weight, and the rocket engine thrust. These are the basic four forces on all rockets.

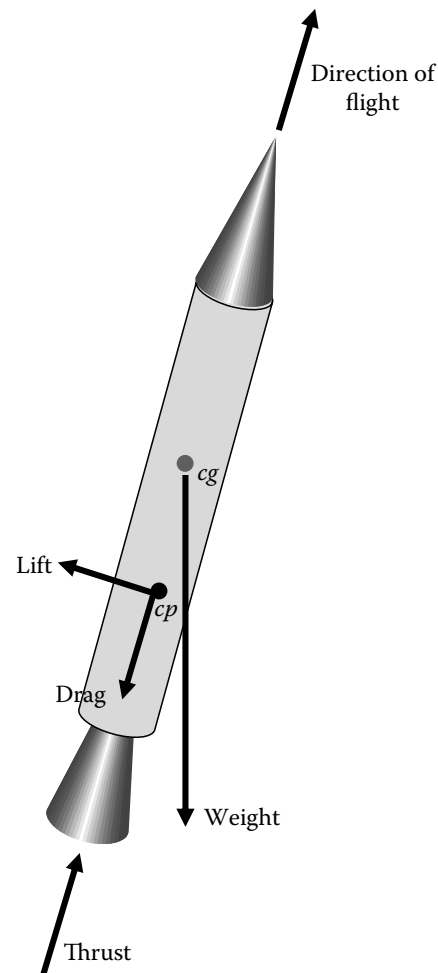


FIGURE 3.10
The forces acting on a rocket.

3.6.1 Aerodynamic Forces

The lift and drag on the rocket vehicle are only applicable if the rocket is in an atmosphere. Note that sometimes the atmosphere can be perceived to be a vacuum when it really isn't. The International Space Station (ISS) is at an orbit of over 400 km and is in a vacuum that is deadly to astronauts without spacesuits. However, the drag on the ISS is appreciable enough even at that orbital altitude that it often has to be reboosted in order to maintain its

orbit. Although the number of molecules of air (atomic oxygen at that altitude mostly) is extremely small, the surface area of the ISS is large enough and the relative velocity between the atmosphere and the spacecraft is fast enough that a significant drag force is imparted to the space station.

Of course, the aerodynamic forces are particular to size and shape of the vehicle as well as the angle of attack of that shape with respect to the direction of motion. The lift force, L , is found by

$$L = C_L A \frac{\rho v^2}{2}. \quad (3.51)$$

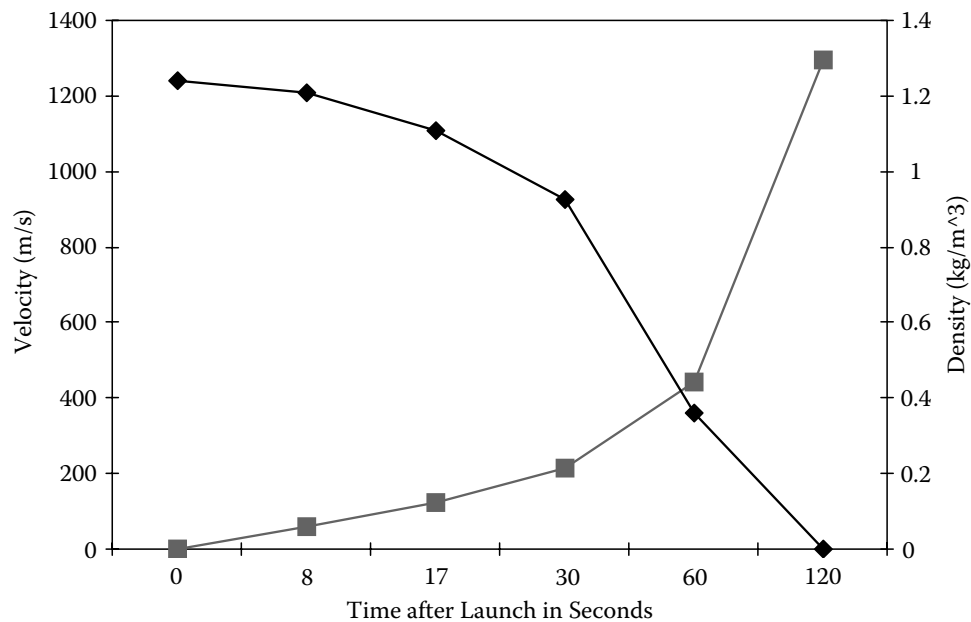
In Equation 3.51, C_L is the lift coefficient, A is the effective area of the surface impacting the air, ρ is the atmospheric density at the altitude of flight, and v is velocity of the rocket.

Similar to the lift, the drag force is calculated as

$$D = C_D A \frac{\rho v^2}{2} \quad (3.52)$$

where C_D is the drag coefficient of the vehicle. We should note that the quantity $\rho v^2/2$ in both of the above equations is known as the dynamic pressure and is often referred to as Q . During a shuttle launch just before the announcement that the shuttle is “go at throttle-up,” it is usually announced that the shuttle has just passed through “max-Q.” Max-Q is the point where the spacecraft has just pushed through the atmosphere at the highest dynamic pressure it will meet during the flight trajectory. Max-Q for the Space Shuttle is at an altitude of about 11 km and at a velocity of about 442 m/sec. Figure 3.11 shows a graph of the velocity of the airflow against the Space Shuttle versus the time of flight. It also shows the density of the atmosphere the Shuttle meets as it flies through its trajectory. Somewhere around 60 seconds, the vehicle goes through max-Q. Figure 3.12 shows the dynamic pressure on the Space Shuttle versus flight time. The peak pressure is around 35,000 Pa (Pascal).

The dynamic pressure due to the aerodynamic forces on the Space Shuttle is the reason that we hear the “go at throttle-up” call around 68 seconds or so. This is because the rocket system is not designed to plow through the atmosphere at maximum velocity prior to this because the max-Q would be too large. It is not necessarily too large for the frame to handle, but throttling more fuel through the engines would be an inefficient use of fuel by pushing harder and harder against the pressure for only slight changes in Δv with huge fuel penalties. Therefore, waiting just a few seconds more at a lower mass flow rate until the dynamic pressure starts dropping off rapidly is a more fuel efficient flight profile. After passing through the max-Q, the rocket then goes full throttle.

**FIGURE 3.11**

The airflow velocity and atmospheric density against the Space Shuttle versus time after launch.

3.6.2 Example 3.3: Drag Force on the Space Shuttle

Given the information in this section, the effective area diameter of the shuttle is about 20 m, and that the drag coefficient, C_D , for the shuttle is about 0.2, calculate the drag force on the vehicle at max-Q.

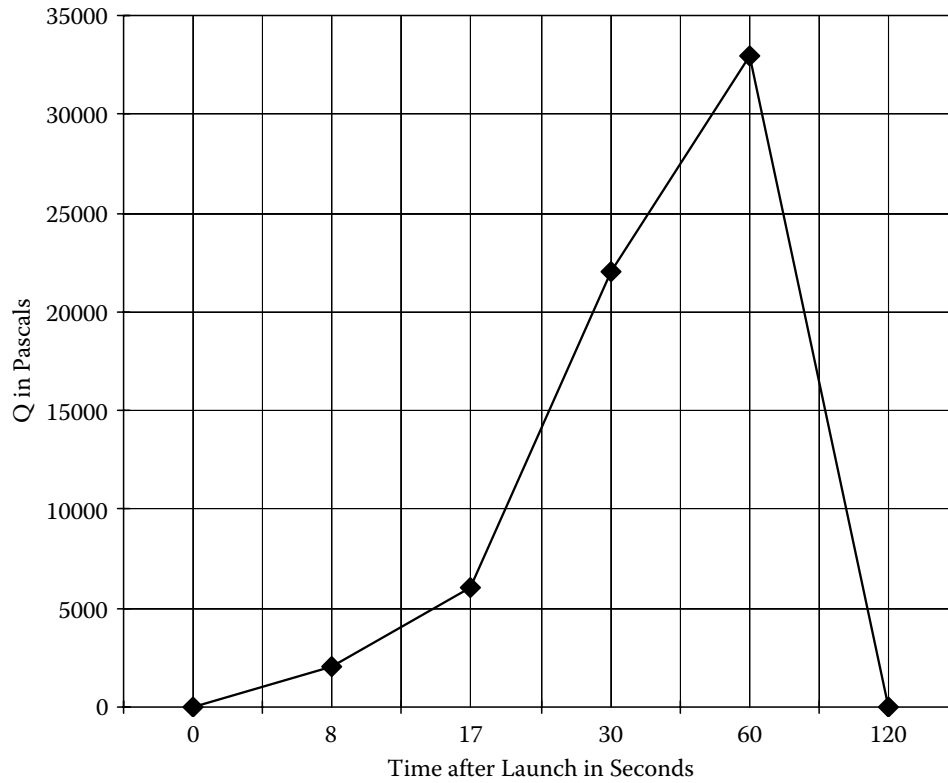
Using Equation 3.52 and substituting in the numerical values, we see

$$D = C_D A \frac{\rho v^2}{2} = 0.2 \pi (10m)^2 (3500Pa) = 2.199 \times 10^6 N \approx 2.2MN. \quad (3.53)$$

3.6.3 Rocket Stability and the Restoring Force

Now that we understand weight, thrust, lift, and drag, we can discuss how they impact the flight dynamics of a rocket in a little more detail. Refer back to Figure 3.10 and the forces on a rocket. The weight is directed downward from the center of gravity of the rocket as shown. The thrust is parallel with the direction of travel of the rocket. The drag is parallel, but in the opposite direction of the thrust. And, the lift forces are typically perpendicular to the flight path unless the rocket structure is of an unorthodox design. Another crucial aspect of the flight dynamics is that the four forces incident on the rocket are changing.

So far we have treated these forces as scalar quantities, but in actuality they are vectors that are changing in direction and magnitude throughout

**FIGURE 3.12**

Dynamic pressure, Q , against the Space Shuttle versus time after launch.

the flight path. As standard practice for understanding forces acting on a body, we sum up the four force vectors to determine the total force and is written as

$$\sum \mathbf{F} = \mathbf{F}_{thrust} + m\mathbf{g} + \mathbf{L} + \mathbf{D}. \quad (3.54)$$

Note that, in this case, the gravitational parameter $\mathbf{g}$ was used as a vector. The direction of $\mathbf{g}$ would be radially inward from the center-of-gravity, cg , (the gray dot in Figure 3.10) of the rocket to the center of the Earth and can be in whatever coordinate system deemed appropriate. Another important parameter mentioned in Figure 3.10 that is key in Equation 3.54, but is not shown, is the angle of displacement, α . The angle of displacement is the angle between the flight path line and the vertical. This is sometimes referred to as just the *displacement angle*.

Again, refer back to Figure 3.10. An important parameter shown as the black dot is the center-of-pressure, cp . The two parameters cg and cp are very important to stable rocket flight and, therefore, deserve some discussion here. We'll start with the cg .

The center of gravity is sometimes referred to as the “center of mass” of a system when talking about two bodies. But, in rocketry and in reference to a rocket vehicle, the *cg* is the term used the most. The *cg* of a solid object (assume uniform, or near uniform density) is defined as average position of all particles making up that object weighted by the masses of the particles and is typically the geometric centroid of the object and is a distance measured in meters. Mathematically speaking the *cg* is

$$cg = \frac{\sum_i w_i d_i}{\sum_i w_i} = \frac{\sum_i m_i g d_i}{\sum_i m_i g} = \frac{\sum_i m_i d_i}{\sum_i m_i} . \quad (3.55)$$

Note that Equation 3.55 shows that the *cg* is the same whether it is measured as the center of weight or the center of mass and, hence, the usage center of gravity makes the most sense. We should also note that without the aid of detailed engineering drawings of all components with known densities it is easier just to weigh the components as opposed to calculating their masses. In some cases it is not.

Figure 3.13 shows a typical liquid fueled rocket lying horizontally with the *cg* marked with a large dot. The components (or particles as from our

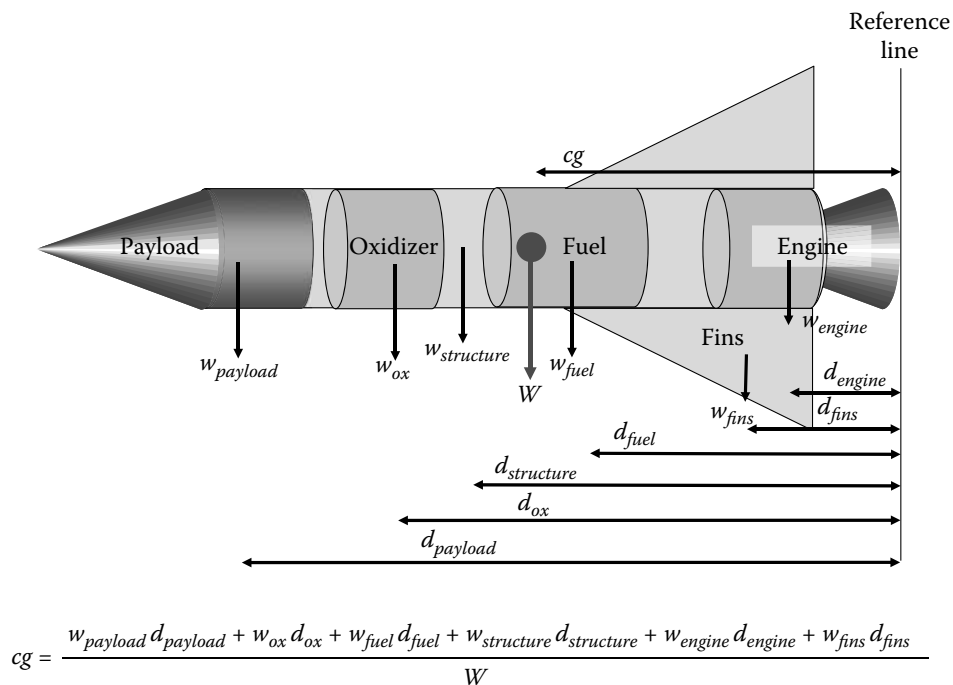


FIGURE 3.13

Center of gravity of a rocket is calculated as shown in the above figure.

definition above) making up the rocket are the payload (in the nose cone), the oxidizer, fuel, structure, engine, and the guidance fins. The total weight, W , of the rocket times the location of the cg measured from a reference line at the rear of the rocket is equal to the sum of the weight of each of the components listed above multiplied by their respective distances from the reference line. Mathematically, the cg times the weight of the rocket is found from the following equation

$$cgW = w_{payload}d_{payload} + w_{ox}d_{ox} + w_{fuel}d_{fuel} + w_{structure}d_{structure} + w_{engine}d_{engine} + w_{fins}d_{fins}. \quad (3.56)$$

Or, from Equation 3.55, we see that

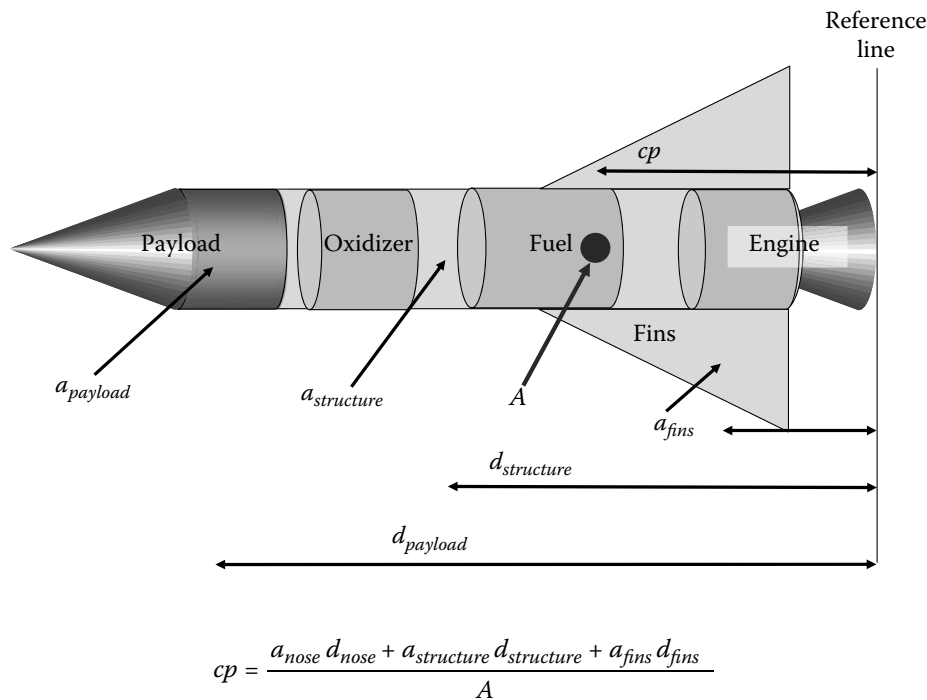
$$cg = \frac{w_{payload}d_{payload} + w_{ox}d_{ox} + w_{fuel}d_{fuel} + w_{structure}d_{structure} + w_{engine}d_{engine} + w_{fins}d_{fins}}{W}. \quad (3.57)$$

Also,

$$cg = \frac{m_{payload}d_{payload} + m_{ox}d_{ox} + m_{fuel}d_{fuel} + m_{structure}d_{structure} + m_{engine}d_{engine} + m_{fins}d_{fins}}{M_{total}}. \quad (3.58)$$

Again, both Equation 3.57 and Equation 3.58 are useful in that sometimes it is easier to weigh the components of the vehicle and sometimes it is not. Think of the Space Shuttle that has over 2,000,000 moving parts. It would take quite some time to weigh all precisely with propellants and lubricants flowing through them. It would also be difficult to raise a vehicle like the Shuttle on a pendulum or balance (like a teeter totter) to determine the cg . Various methods for calculating the cg for such vehicles are used that include some measurements as well as detailed engineering solid modeling. Yet, another tool needed for modern rocket experts is that of three-dimensional (3D) solid modeling on high performance computers.

Let us talk now about the center of pressure. The cp is the location on a solid body where the sum of all the forces from pressure fields (in our case, mostly aerodynamic pressure fields) act while creating no moment about that location. In other words, the cp of a solid object with a total surface area, A , is defined as the average position of all particles having an exterior surface areal component making up that object weighted by the area of the individual particles and is a distance measured in meters. The cp is shown as the large dot in Figure 3.14. Mathematically speaking, the cp is

**FIGURE 3.14**

Center of pressure of a rocket is calculated as shown in the above figure.

$$cp = \frac{\sum_i a_i d_i}{\sum_i a_i} = \frac{\sum_i a_i d_i}{A} . \quad (3.59)$$

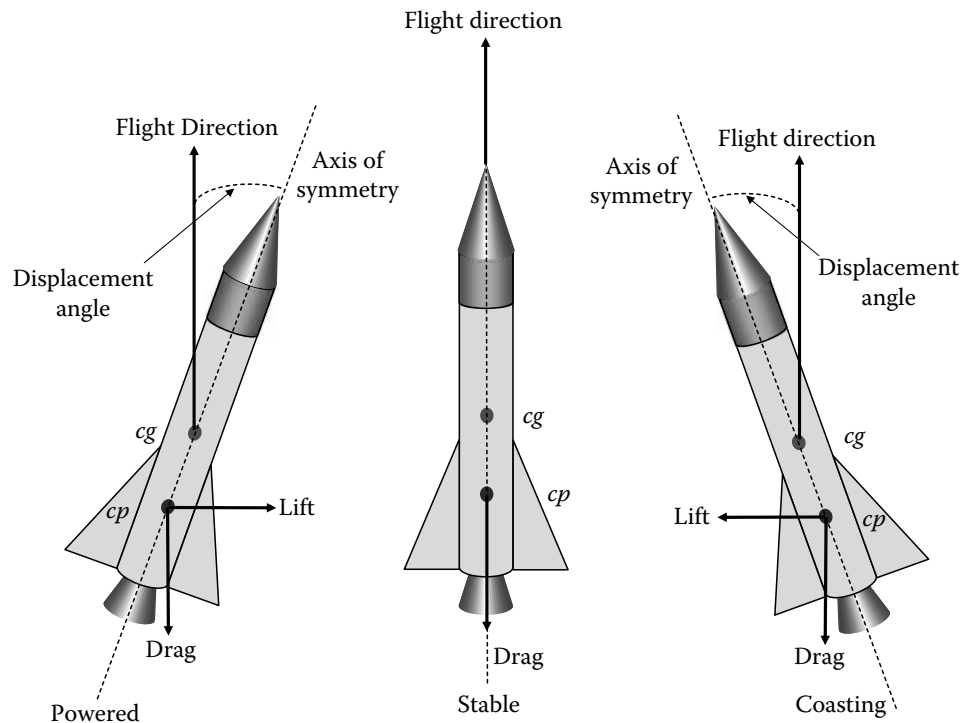
Like Equation 3.57 and Equation 3.58, it can be mathematically calculated via the following equation

$$cp = \frac{a_{nose} d_{nose} + a_{structure} d_{structure} + a_{fins} d_{fins}}{A} . \quad (3.60)$$

One other note here is that in some reference the cp is measured from the nose of the vehicle as opposed to the same reference line as the cg . In this text, we will always use the same reference line (the rear- or bottom-most point on the rocket) as the reference line for both parameters.

Now that we have defined cg and cp we can continue with our development of rocket flight stability. Figure 3.15 shows three flight conditions of a rocket in flight. These conditions are *powered*, *stable*, and *coasting*.

In the real world, there are things that cannot always be accounted for, such as wind gusts, shear forces, pings from flying foam insulation debris,

**FIGURE 3.15**

Three modes of rocket flight stability are illustrated.

geese, lightning, and any myriad things that can perturb the rocket's flight path. These external forcing functions add what is sometimes referred to as "wobble" to the vehicle. This wobble is simply a perturbation to the normal attitude vector of the rocket. If such a perturbation occurs during *powered* flight (while the engines are thrusting), the rocket will be displaced by some displacement angle and the vehicle, while still traveling along the same flight path, will not be aligned in the optimum aerodynamic configuration. The lift and drag forces of the rocket will increase creating a torque about the *cg*. The same happens if the rocket is in the *coasting* mode (the engines are off) except the lift and drag forces create a torque in the opposite direction.

A very important aspect of rocket design is that the *cp* MUST be below the *cg* and here is why. If the *cp* is below the *cg*, then the torques created by the perturbing forces during both *powered* and *coasting* flight will be in directions that self-correct the perturbation and place the rocket back into a properly optimized aerodynamic attitude. This is known as the *restoring force* and this force puts the rocket back into the *stable* flight mode.

If, for some reason, the *cp* is above the *cg*, then any perturbing forces will create torques that have a destabilizing effect and will send the rocket spinning madly out of control. Any true model rocket enthusiast has seen this at least once during his or her life. A nose cone that is too heavy can get a very slight perturbation and cause the rocket to fall head over heels,

so to speak. At that point, the cg is above the cp and the model rocket, which is still perturbed, will spin back over to the point with the top-heavy nose cone on top. Still under the perturbations of spin it will fall over once again placing the cp on top and so on until the rocket goes careening into oblivion or atop the neighbor's roof. Either case is not an optimum *stable* flight attitude.

Some missile systems in use today still use simple solid motors with aerodynamic stability systems (fins). It is important in the design phase of these systems to heed the above section closely. However, most modern larger rocket systems do not rely on aerodynamic-driven control systems. In fact, most modern rockets adjust the attitude by tilting the thrust of the main engine. This technique is known as *thrust vector control* and is what is happening when we see the nozzles of launch vehicles moving around at the base of the rocket. This is also the reason why most modern launch vehicles do not need fins.

3.6.4 Rocket Attitude Control Systems

Figure 3.16 shows the four basic types of systems for *attitude control* for rockets. They are moveable aerodynamic structures, such as fins, gimbaled thrust, vernier thruster rockets, and thrust vanes. Moveable aerodynamic structures, such as fins, function in the same way as ailerons and rudders on aircraft and require the rocket to be in an atmosphere that is dense enough for the control surface to be of any use. The other three methods, gimbaled

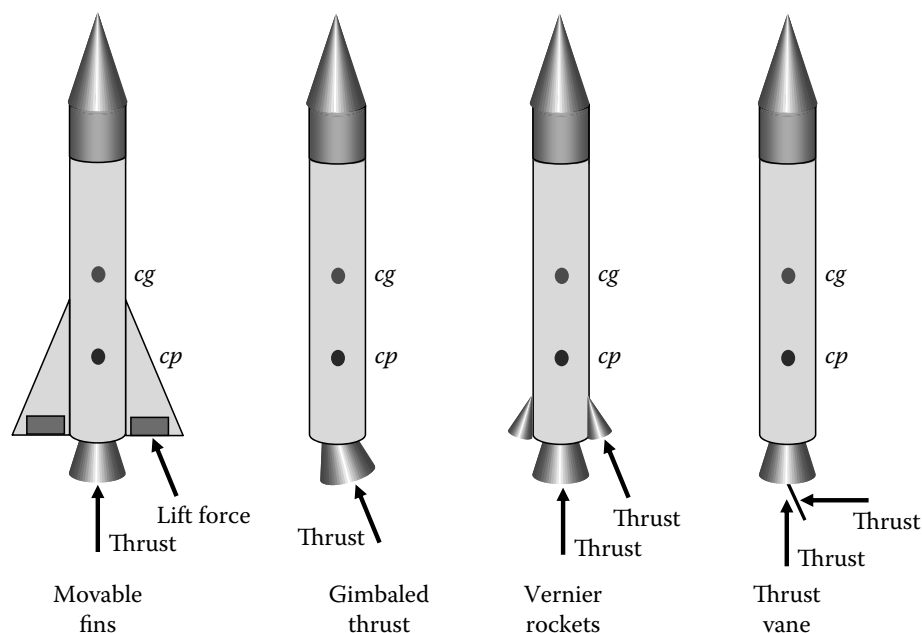


FIGURE 3.16

Pictured are four types of rocket attitude control.

thrust, vernier thruster rockets, and thrust vanes, are all variations on the same technique known as *thrust vector control* or TVC.

TVC works by actually redirecting the thrust vector either by swiveling the main engine nozzles, using smaller vernier rockets to thrust with a desired vector, or moving a vane in front of the main engine thrust to redirect it.

Gimbaled thrust is the technique employed on the Space Shuttle Orbiter. The SSMEs of the system as well as the nozzles of the solid rocket boosters (SRBs) are gimbaled and are used throughout the Earth-to-orbit phase to correct and optimize the flight trajectory. The Soyuz rocket uses combinations of aerodynamic fins and vernier thrusters. The Shuttle Orbiter actually has vernier rockets in the nose in order to adjust attitude while in orbit. The Mercury, Gemini, and Apollo spacecraft also employed vernier thrusters for in-space maneuvering and control. The German V2 rockets of the World War II era used thrust vanes for control. Thrust vanes are typically seen on modern fighter jets and have also been implemented on some experimental rocket vehicles.

3.6.5 8 Degrees of Freedom

Figure 3.17 shows a rocket in flight along the x -axis. If the rocket spins about the x -axis, this is called a *roll* or a *rolling maneuver*. If the rocket rotates its nose up or down and, therefore, spins about the y -axis, this is called a *pitch* or a *pitching maneuver*. If the rocket rotates the nose toward the y -axis or the negative y -axis about the z -axis, this is called a *yaw* or *yaw maneuver*. The rocket can also have motion in the forward or backward directions along the x -axis due to drag or thrust. We have just described 8 degrees of freedom of motion for

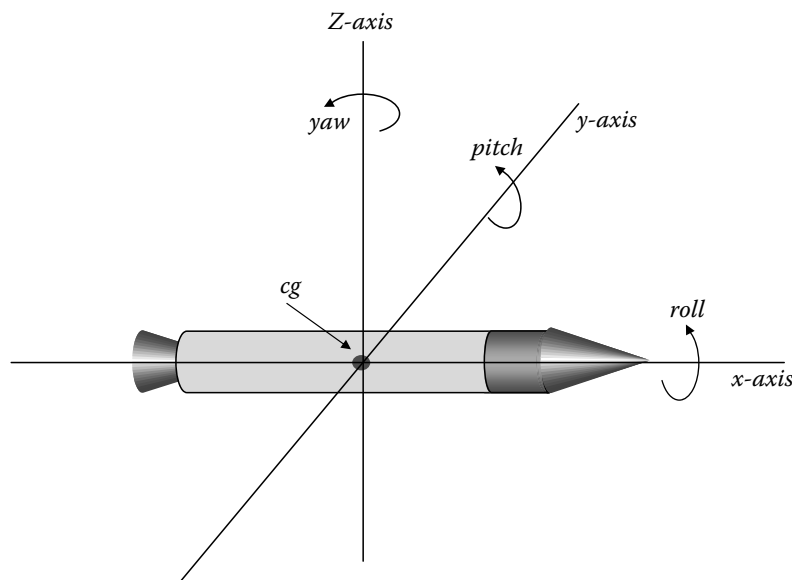


FIGURE 3.17

Rocket roll, pitch, and yaw maneuvers are shown.

the rocket. Rocket scientists and engineers will often refer to this as 8-DOF of motion. Sometimes we will hear the term 6-DOF, but this is when drag and thrust are neglected. So, the 8-DOF dynamics can be described as:

- Positive roll, θ_x
- Negative roll, $-\theta_x$
- Positive pitch, θ_y
- Negative pitch, $-\theta_y$
- Positive yaw, θ_z
- Negative yaw, $-\theta_z$
- Forward thrust, Δx
- Drag or negative thrust, $-\Delta x$

In order for the rocket to correct for perturbations and disturbances along its flight path, attitude corrections within these 8-DOFs must be continually made. Accomplishing this controlled flight is quite an endeavor.

Figure 3.18 shows a typical attitude control system (ACS). Note that there is a different circuit for each axis. This is because the control for each axis can be separated from the others simplifying the dynamics and complexity of the ACS itself. Do note, however, that there are inputs from each of the other two axes into the disturbance torque to account for any errors that an attitude correction for one axis might induce on another.

The initial state of the rocket is input into the circuit and is compared to the attitude from sensor data giving an attitude error value. Then a control processor (computer) takes the difference data (error in attitude) and calculates if there are attitude correction thrusts that need to be generated. Also, at this point the command unit can input other attitude maneuver commands into the system. Here is where a pilot's input from a joystick might come into play.

The commands for correction thrusts are then sent to the ACS thrusters, which fire for the calculated amount of time and with the appropriate force. Then the control actions and external forcing disturbances move the rocket vehicle in space as well as bending, flexing, and sloshing components of the

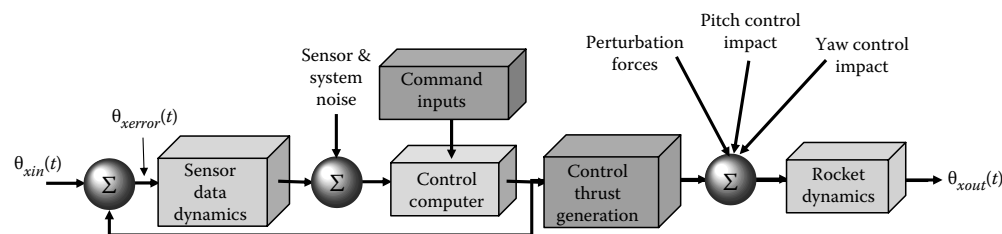


FIGURE 3.18

An image of the attitude control circuit for a rocket system. Note that the diagram is only for the roll control of the vehicle.

rocket. The rocket achieves a new state of attitude, which is then fed back into the initial state input side of the circuit for the process to start all over. The controller circuit will determine through the same process if the external forces and the correction thrusts placed the rocket in the optimum attitude and will decide if too great or too little correction thrusts were made. This is a continuous process as long as the rocket is in flight.

Though it is beyond the scope of this text to develop the control algorithm in detail for a rocket vehicle, we can discuss it in general. The ACS control circuit shown in Figure 3.18 implements what is known as a *proportional, integral, derivative* (PID) controller. The open loop PID control circuit is described mathematically as

$$\theta_{out}(t) = K_p \theta_{in}(t) + K_i \int_0^t \theta_{in}(t) dt + K_d \frac{d\theta_{in}(t)}{dt}. \quad (3.61)$$

Here K_p is the proportional gain, K_i is the integral gain, and K_d is the derivative gain. The proportional gain is equivalent to a thrust in one direction, which would lead to the rocket attitude to vibrate like an undamped mechanical spring. It is this proportional gain that determines the largest corrections. For example, if the attitude is incorrect by 7° then the proportional controller sends a signal to the thrusters to correct for 7° . The derivative control component adds a damping thrust. It determines the rate at which a thrust should decrease or increase to damp out the disturbance. The integral controller looks at a longer period of time of the changing attitude and looks for longer acting attitude errors. These errors are usually instantaneously small, but can cause large course errors over time. The integral component is needed as a check and balance to each of the other more abrupt control components to maintain flight path accuracy.

It should also be noted that sometimes in the literature the PID controller is also called a *position, integral, derivative* controller. Due to the large mechanical forces involved with rocket systems, the PID controller is quite ideally suited for the task of rocket ACS.

It is beyond the scope of this text to develop in detail a complete closed feedback loop model for a rocket's attitude control system. However, a typical system follows the math of a damped oscillator similar to a damped mechanical spring. A general solution for such a system is

$$\theta_{out}(t) = \theta_{in} - \theta_{in} e^{-\zeta \omega t} \cos(\omega t + \beta). \quad (3.62)$$

Here θ_{in} is the input state of the controller or the initial condition of the attitude of the rocket, ζ is the damping coefficient, ω is the frequency of the system, and β is the center frequency of the bandwidth of the system. With

a much more detailed analysis of a rocket system, these coefficients can be solved in terms of the PID coefficients and the equation might vary in complexity from system to system, but developing that solution is unnecessary here as we are merely trying to get an idea of how the controller for the ACS works. Complete textbooks and doctoral dissertations are written on the complex issues involving detailed ACS.

Figure 3.19 shows a graph of Equation 3.62 for several combinations of values for the control equation coefficients. Notice that it takes a certain amount of interplay between all three of them for the system to be stable where the oscillations are damped completely out. As an exercise for the reader, model Equation 3.62 in a math-modeling software package like Mathcad and compare outputs due to different values for the three constants. When the right gains are used, the rocket's many degrees of attitude freedom can be controlled.

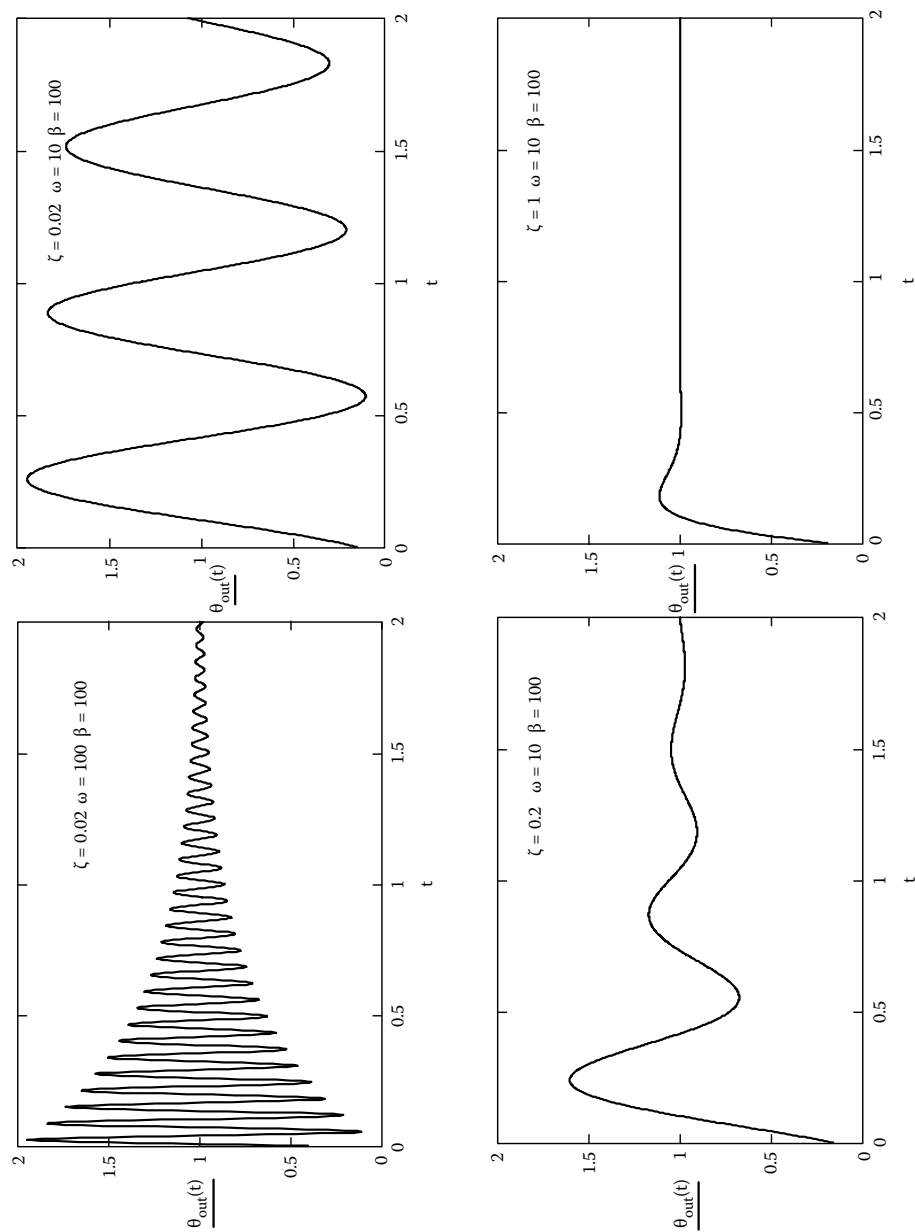
3.7 Chapter Summary

In this chapter, we have learned a great deal about how rockets work. We started in Section 3.1 and developed the concept of thrust from Newton's laws of motion. From the laws of motion, we developed definitions for mass flow rate and how this impacts rockets through throttling. And we found the rocket thrust equation, which is a good tool that is useful for designing a rocket engine for a particular mission.

The derivation of the thrust led us to understanding another key parameter for rocket engine design and that is the specific impulse. In Section 3.2, we developed the calculation for specific impulse and we discussed how this parameter is important when describing the efficiency of a rocket engine.

Likewise, the final design parameter for rocket engines discussed in this chapter was developed in Section 3.3. The weight flow rate was discussed and it is clear now that with the thrust, specific impulse, and the weight flow rate a rocket scientist or engineer has most of the information needed to understand the capabilities of a given rocket engine. These are also important tools in the first steps of mission design. Knowing the type of mission tells the engineer if a high thrust is needed, such as for a launch vehicle, or if a high specific impulse is needed as in interplanetary missions. The weight flow rate then tells the designer something about the actual physical size needed for the rocket engine to achieve the desired thrust and/or specific impulse.

In Section 3.4, we found the famous Tsiolkovsky rocket equation. This is the bread and butter for rocket scientists and engineers. The rocket equation allows us to understand how a rocket functions over a complete flight trajectory from the beginning of the flight when its fuel tanks are full to the end of

**FIGURE 3.19**

The image shows an attitude control circuit output as a function of time for various values of the control parameter coefficients. Ideal response would be immediately settling with no ringing.

it when all the propellant has been burned up and the tanks are empty. The equation tells us that there is a ratio of full-to-empty mass of the rocket that is the key parameter in determining how much Δv the rocket can supply to a payload. And finally, the equation leads us to realize that in some cases it is better to have multiple stages of rocket engines for a more efficient system design. Staging was discussed in detail in Section 3.5.

To complete our discussion of how rockets work, we discussed in Section 3.6 the flight dynamics and how to control the attitude of a rocket vehicle during flight. We developed important concepts of rocket make-up including the center-of-gravity and the center-of-pressure. We showed how to calculate these parameters and why they are important to rocket scientists and engineers. Then we developed the actual process for controlling the rocket's attitude during flight and discussed the PID controller.

From the basics of thrust to the complexity of ACS, we now have a basic understanding of rocket vehicles and rocket flight. From Chapter 1, Chapter 2, and now Chapter 3, we are beginning to see how intricately detailed and massively complex the field of rocket science and engineering has become. And we have yet to discuss combustion chambers and nozzles in any detail. That will come in the next chapter.

Exercises

- 3.1 What do you call the opening at the bottom of the combustion chamber?
- 3.2 Highly accelerated exhaust gases leaving the rocket engine nozzle propel the spacecraft through which of Newton's Laws of Motion?
- 3.3 Why is $m\text{-dot}$ important to the astronaut phrase "throttle-up"?
- 3.4 What does "throttle-up" mean?
- 3.5 What is the *impulse momentum theorem* and what does it tell us?
- 3.6 What is the difference between the *effective exhaust velocity* and the *equivalent velocity*?
- 3.7 Define *equivalent velocity*.
- 3.8 Define *specific impulse*.
- 3.9 What is the importance of the *weight flow rate*?
- 3.10 What are three key parameters for rocket engine design?
- 3.11 What is the *propellant mass ratio*? What else is it sometimes called?
- 3.12 What is *hybrid staging*?
- 3.13 What are three types of staging?
- 3.14 Describe the four major subsystems of a rocket.

- 3.15 What is max-Q?
- 3.16 Why do most launch vehicles wait until after max-Q to “go at throttle-up”?
- 3.17 What are three rocket flight conditions?
- 3.18 What is the *restoring force*?
- 3.19 Discuss four types of attitude correction systems.
- 3.20 What is a PID controller?
- 3.21 Given a rocket nozzle with an exit area of 1 m^2 and an exit pressure of $101,325 \text{ Pa}$, what is the force on the nozzle due to the pressure difference inside and outside the rocket if the rocket is at sea level?
- 3.22 In Exercise 3.21, calculate the force on the nozzle if the rocket is in space and the pressure outside the rocket is zero.
- 3.23 In Exercises 3.21 and 3.22, determine the force on the rocket if the $\dot{m}$ of the engine is 1 kg/sec and the exhaust velocity is 400 m/sec .
- 3.24 A rocket engine has an I_{sp} of 363 sec and can produce a thrust of 2 MN . Calculate the equivalent velocity for the engine.
- 3.25 In Exercise 3.24, determine the $\dot{m}$ of the engine.
- 3.26 In Exercises 3.24 and 3.25, determine the *mass ratio* required to reach a Δv of $7,700 \text{ m/sec}$.
- 3.27 In Exercises 3.24 to 3.26, determine the burn time required to achieve the Δv of $7,700 \text{ m/sec}$ assuming the *mass ratio* calculated in Exercise 3.26.
- 3.28 Given a two-stage launch vehicle with an engine that produces an $I_{sp} = 400 \text{ sec}$, a payload mass of $10,000 \text{ kg}$, stage 1 structure mass of $10,000 \text{ kg}$, stage 2 structure mass of $10,000 \text{ kg}$, determine the mass ratio and the total mass of propellant required to reach LEO. Assume the total Δv required is $7,700 \text{ m/sec}$. Determine the Δv after each stage and the propellant mass for each stage.
- 3.29 Assume that the drag coefficient for the ISS is 0.2 and its velocity is $27,744 \text{ km/h}$. The density of the atmosphere at ISS's orbit is about $1 \times 10^{-11} \text{ kg/m}^3$. If the surface area of the ISS is about $3,000 \text{ m}^2$, what is the drag force?
- 3.30 Consider three blocks of density 1 kg/m^3 . Block 1 is 1 m per side in dimension. Block 2 is 2 m per side in dimension. Block 3 is 3 m per side in dimension. The blocks are oriented in such that the largest block, Block 3, is on the bottom. Block 2 is then stacked on Block 3, and then Block 1 is stacked on Block 2. The faces of the blocks are aligned and the center of each the blocks make a straight line upward through them. Figure 3.20 shows the blocks and how they are stacked. With the bottom of the stack as the reference line, calculate the center-of-gravity of the stack of blocks.

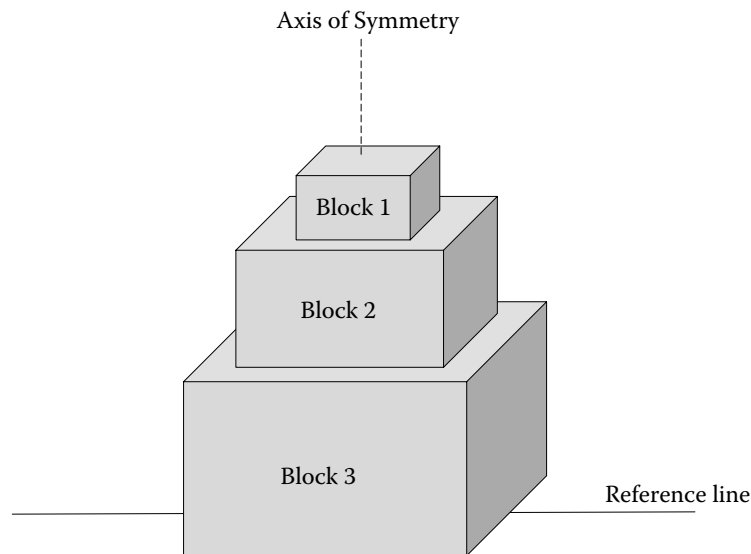
**FIGURE 3.20**

Diagram of stacked blocks for Exercise 3.30.

- 3.31 In Exercise 3.29, if Block 1 was twice as tall and the three blocks remain in the same stacked configuration, calculate the center-of-gravity.
- 3.32 In Exercise 3.29, calculate the center-of-pressure for the stacked blocks.
- 3.33 In Exercise 3.30, calculate the center-of-pressure for the stacked blocks.
- 3.34 Define 8-DOF and explain each component in detail.

16. Introduction to Astrodynamics

near the horizon is almost as easy as pointing to a target or ground antenna at nadir. In this case the working limit on the spacecraft's elevation angle depends on other factors, such as the transmission of the atmosphere for a selected wavelength or possible obstruction by local geography. For example, ground stations ordinarily limit their elevation to approximately 5 deg above the horizon because of the reduced atmospheric transmission at lower elevation angles.

Pointing requirements normally arise from the spacecraft's housekeeping functions or from the need to point a particular instrument or antenna toward a ground target. Housekeeping requirements such as solar array pointing and orbit maneuvers ordinarily demand pointing accuracies of 0.25 to 1 deg. Consequently, for most missions, the need to point the mission sensor or antenna is more important. Here again two cases exist. If we wish to point the sensor at a single target, then we will generally try to point the center of the sensor at the target. By doing so, we establish a pointing requirement to place the target within the field of view. Specifically, if the payload sensor's field of view is four times the 3σ pointing error, then the target will lie within the field of view with a 6 σ probability, or virtual certainty. For example, if the FireSat sensor has a 1 deg square field of view, an overall pointing requirement of 0.25 deg will assure that the target will be within the field of view.

In pointing we may also want to eliminate overlapping coverage. For example, if we wish to take a series of pictures, we must overlap the pictures by more than the pointing error to ensure continuous coverage of the ground. This requirement, in turn, implies that the spacing between pictures must equal the field-of-view size less the pointing error. Thus, with a large pointing error, we must accept having fewer pictures at a given time and increased resource costs in terms of time, power, and data rate for a given level of coverage. It is common to have a pointing accuracy of 10% to 20% of the field-of-view diameter. Driving the pointing under 10% of the field-of-view diameter will only slightly improve overall coverage. On the other hand, a pointing error worse than 20% of the field-of-view size can require substantial overlap, thus greatly diminishing the overall system's coverage and resource utilization.

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Chapter 6

Introduction to Astrodynamics

Daryl G. Boden, United States Naval Academy

- 6.1 Keplerian Orbits
Satellite Equations of Motion; Constants of Motion; Classical Orbital Elements; Satellite Ground Tracks; Time of Flight in an Elliptical Orbit; Orbit Determination
- 6.2 Orbit Perturbations
Third-Body Perturbations; Perturbations Because of a Nonspherical Earth; Perturbations from Atmospheric Drag; Perturbations from Solar Radiation
- 6.3 Orbit Maneuvering
Coplanar Orbit Transfers; Orbit Plane Changes; Orbit Rendezvous
- 6.4 Launch Windows
- 6.5 Orbit Maintenance

Astrodynamics is the study of a satellite's *trajectory* or *orbit*, that is, its path through space. The satellite's *ephemeris* is a table listing its position as a function of time. The first section below explains the terms used to describe satellite orbits, provides equations necessary to calculate orbital elements from position and velocity, and shows how to predict the future position and velocity of a satellite. We base this method on a simple, but accurate, model treating the Earth and the satellite as spherical masses. The next section discusses how forces other than the Newtonian gravitational force affect the orbit of a satellite. The third section explains maneuvering strategies used to change the satellite's orbit. The final two sections discuss the available launch times and methods for maintaining satellite orbits.

Several textbooks are available in the areas of satellite orbits and celestial mechanics. Some of the most popular are Bate, Mueller, and White [1971], Battin [1999], Danby [1962], Escobal [1965], Kaplan [1976], and Roy [1991]. More recently Chobotov [1996], Wiesel [1996], and Vallado [1997] have provided works specifically on spacecraft astrodynamics.

6.1 Keplerian Orbits

Explaining the motion of celestial bodies, especially the planets, has challenged observers for many centuries. The early Greeks attempted to describe the motion of celestial bodies about the Earth in terms of circular motion. In 1543, Nicolaus Copernicus proposed a heliocentric (Sun-centered) system with the planets following circular orbits. Finally, with the help of Tycho Brahe's observational data, Johannes Kepler described elliptical planetary orbits about the Sun. Later, Isaac Newton provided a mathematical solution for this system based on an inverse-square gravitational force.

Kepler spent several years reconciling the differences between Tycho Brahe's careful observations of the planets and their predicted motion based on previous theories. Having found that the data matched a geometric solution of elliptical orbits, he published his first two laws of planetary motion in 1609 and his third law in 1619. Kepler's three laws of planetary motion (which also apply to satellites orbiting the Earth) are:

- **First Law:** The orbit of each planet is an ellipse, with the Sun at one focus.
- **Second Law:** The line joining the planet to the Sun sweeps out equal areas in equal times.
- **Third Law:** The square of the period of a planet is proportional to the cube of its mean distance from the Sun.

6.1.1 Satellite Equations of Motion

Figure 6-1 depicts the key parameters of an elliptical orbit. The *eccentricity*, e , of the ellipse (not shown in the figure) is equal to c/a and is a measure of the deviation of the ellipse from a circle.

Isaac Newton explained mathematically why the planets (and satellites) follow elliptical orbits. Newton's *Second Law of Motion*, applied to a constant mass system and, combined with his *Law of Universal Gravitation*, provides the mathematical basis for analyzing satellite orbits. Newton's law of gravitation states that any two bodies attract each other with a force proportional to the product of their masses and inversely proportional to the square of the distance between them. The equation for the magnitude of the force caused by gravity is

$$F = -GMm/r^2 \quad (6-1)$$

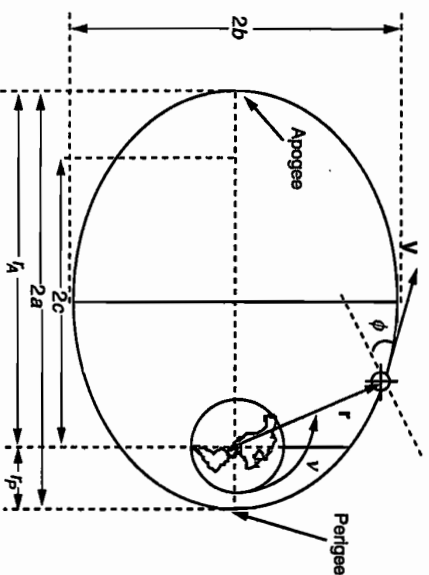
$$\equiv -\mu m/r^2$$

where F is the magnitude of the force caused by gravity, G is the universal constant of gravitation, M is the mass of the Earth, m is the mass of the satellite, r is the distance from the center of the Earth to the satellite, and $\mu \equiv GM$ is the Earth's gravitational constant ($= 398,600.5 \text{ km}^3\text{s}^{-2}$).

Combining Newton's second law with his law of gravitation, we obtain an equation for the acceleration vector of the satellite:

$$\ddot{\mathbf{r}} + (\mu/r^3)\mathbf{r} = \mathbf{0} \quad (6-2)$$

This equation, called the *two-body equation of motion*, is the relative equation of motion of a satellite position vector as the satellite orbits the Earth. In deriving it, we assumed that gravity is the only force, the Earth is spherically symmetric, the Earth's mass is much greater than the satellite's mass, and the Earth and the satellite are the only two bodies in the system.



- $\mathbf{r}$: position vector of the satellite relative to Earth's center
- $\mathbf{V}$: velocity vector of the satellite relative to Earth's center
- ϕ : flight-path-angle, the angle between the velocity vector and a line perpendicular to the position vector
- a : semimajor axis of the ellipse
- b : semiminor axis of the ellipse
- c : the distance from the center of the orbit to one of the foci
- v : the polar angle of the ellipse, also called the *true anomaly*, measured in the direction of motion from the direction of perigee to the position vector
- r_a : radius of apogee, the distance from Earth's center to the farthest point on the ellipse
- r_p : radius of perigee, the distance from Earth's center to the point of closest approach to the Earth

Fig. 6-1. Geometry of an Ellipse and Orbital Parameters.

A solution to the two-body equation of motion for a satellite orbiting Earth is the *polar equation of a conic section*. It gives the magnitude of the position vector in terms of the location in the orbit,

$$r = a(1 - e^2)/(1 + e \cos v) \quad (6-3)$$

where a is the semimajor axis, e is the eccentricity, and v is the polar angle or true anomaly.

A *conic section* is a curve formed by the intersection of a plane passing through a right circular cone. As Fig. 6-2 shows, the angular orientation of the plane relative to the cone determines whether the conic section is a *circle*, *ellipse*, *parabola*, or *hyperbola*. We can define all conic sections in terms of the eccentricity, e , in Eq. (6-3) above. The type of conic section is also related to the semimajor axis, a , and the specific mechanical energy, ϵ . Table 6-1 shows the relationships between energy, eccentricity, and semimajor axis and the type of conic section.

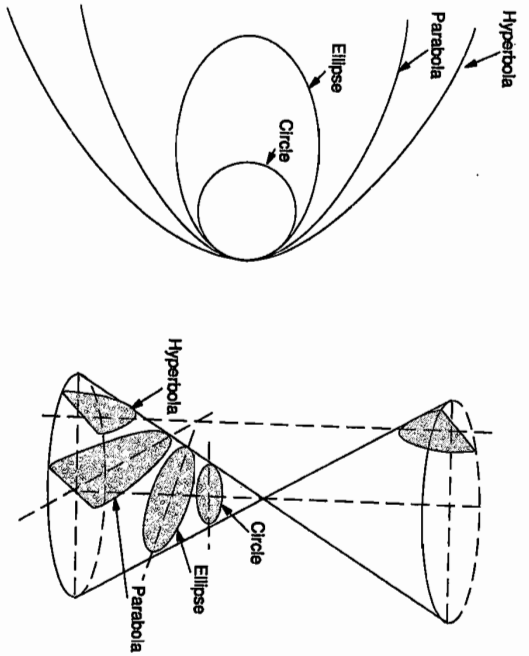


Fig. 6-2. Geometric Origin of Conic Sections. Satellite orbits can be any of four conic sections: a circle, an ellipse, a parabola, or a hyperbola.

TABLE 6-1. Conic Sections. See text for discussion.

Conic	Energy, $\mathcal{E}$	Semimajor Axis, a	Eccentricity, e
Circle	< 0	= radius	0
Ellipse	< 0	> 0	$0 < e < 1$
Parabola	0	∞	1
Hyperbola	> 0	< 0	> 1

6.1.2 Constants of Motion

Using the two-body equation of motion, we can derive several constants of motion of a satellite orbit. The first is

$$\mathcal{E} = v^2 / 2 - \mu / r = -\mu / (2a) \quad (6-4)$$

where $\mathcal{E}$ is the total *specific mechanical energy*, or mechanical energy per unit mass, for the system and is the sum of the kinetic energy per unit mass and potential energy per unit mass. We refer to Eq. (6-4) as the *energy equation*. Because the forces in the system are conservative, the energy is a constant. The term for potential energy, $-\mu/r$, defines the potential energy to be zero at infinity and negative at any radius less than infinity. Using this definition, the specific mechanical energy of elliptical orbits will always be negative. As the energy increases (approaches zero), the ellipse gets larger, and the elliptical trajectory approaches a parabolic trajectory. From the energy Eq. (6-4), we find that a satellite moves fastest at perigee of the orbit and slowest at apogee.

We also know that for a circle the semimajor axis equals the radius, which is constant. Rearranging the energy equation, we find the velocity of a satellite in a circular orbit.

$$\begin{aligned} v_{cr} &= (\mu / r)^{1/2} \\ &\cong 7.905 \, 366 \, (R_E / r)^{1/2} \\ &\cong 631.3481 \, r^{-1/2} \end{aligned} \quad (6-5)$$

where v_{cr} is the circular velocity in km/s, R_E is the Earth's radius in km, and r is the orbit's radius in km.

From Table 6-1, the energy of a parabolic trajectory is zero. A parabolic trajectory is one with the minimum energy needed to escape the gravitational attraction of Earth. Thus, we can calculate the velocity required to escape from the Earth at any distance, r , by setting energy equal to zero in Eq. (6-4) and solving for velocity.

$$\begin{aligned} v_{esc} &= (2\mu / r)^{1/2} \\ &\cong 11.179 \, 88 \, (R_E / r)^{1/2} \\ &\cong 892.8611 \, r^{-1/2} \end{aligned} \quad (6-6)$$

where v_{esc} is the escape velocity in km/s, and r is in km.

Another constant of motion associated with a satellite orbit is the *specific angular momentum*, h , which is the satellite's total angular momentum divided by its mass. We can find it from the cross product of the position and velocity vectors.

$$h = r \times v \quad (6-7)$$

We find that from Kepler's second law, the angular momentum is constant in magnitude and direction for the two-body problem. Therefore, the orbital plane defined by the position and velocity vectors must remain fixed in inertial space.

6.1.3 Classical Orbital Elements

When solving the two-body equations of motion, we need six constants of integration (initial conditions) for the solution. Theoretically, we could find the three components of position and velocity at any time in terms of the position and velocity at any other time. Alternatively, we can completely describe the orbit with five constants and one quantity which varies with time. These quantities, called *classical orbital elements*, are defined below and are shown in Fig. 6-3. The coordinate frame shown in the figure is the geocentric inertial frame,* or GCI, defined in Chap. 5 (see Table 5-1). Its origin is at the center of the Earth, with the X-axis in the equatorial plane and pointing to the vernal equinox. Also, the Z-axis is parallel to the Earth's spin axis (the North Pole), and the Y-axis completes the right-hand set in the equatorial plane. The classical orbital elements are:

- a: *semimajor axis*: describes the size of the ellipse (see Fig. 6-1).
- e: *eccentricity*: describes the shape of the ellipse (see Fig. 6-1).

* A *sufficiently inertial coordinate frame* is a coordinate frame that we can consider to be non-accelerating for the particular application. The GCI frame is sufficiently inertial when considering Earth-orbiting satellites, but is inadequate for interplanetary travel because of its rotational acceleration around the Sun.

i: **inclination**: the angle between the angular momentum vector and the unit vector in the Z -direction.

Ω : **right ascension of the ascending node**: the angle from the vernal equinox to the ascending node. The ascending node is the point where the satellite passes through the equatorial plane moving from south to north. Right ascension is measured as a right-handed rotation about the pole, Z .

ω : **argument of perigee**: the angle from the ascending node to the eccentricity vector measured in the direction of the satellite's motion. The eccentricity vector points from the center of the Earth to perigee with a magnitude equal to the eccentricity of the orbit.

ν : **true anomaly**: the angle from the eccentricity vector to the satellite position vector, measured in the direction of satellite motion. Alternately, we could use **time since perigee passage**, T .

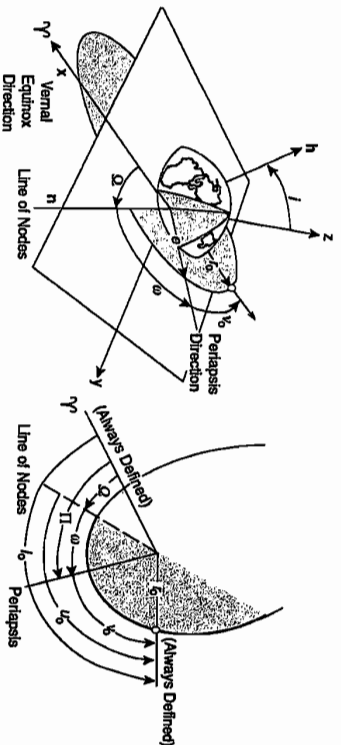


Fig. 6-3. Definition of the Keplerian Orbital Elements of a Satellite in an Elliptic Orbit. We define elements relative to the GCI coordinate frame.

Given these definitions, we can solve for the elements if we know the satellite's position and velocity vectors. Equations (6-4) and (6-7) allow us to solve for the energy and the angular momentum vector. An equation for the **nodal vector**, $\mathbf{n}$, in the direction of the ascending node is

$$\mathbf{n} = \mathbf{Z} \times \mathbf{h} \quad (6-8)$$

We can calculate the eccentricity vector from the following equation:

$$\mathbf{e} = (1/\mu)(V^2 - \mu/r)\mathbf{r} - (\mathbf{r} \cdot \mathbf{v})\mathbf{v} \quad (6-9)$$

Table 6-2 lists equations to derive the classical orbital elements and related parameters for an elliptical orbit.

Equatorial ($i = 0$) and circular ($e = 0$) orbits demand alternate orbital elements to solve the equations in Table 6-2. These are shown in Fig. 6-3. For equatorial orbits, a single angle, Ω , can replace the right ascension of the ascending node and argument of perigee. Called the **longitude of perigee**, this angle is the algebraic sum of Ω and ω . As i approaches 0, Ω approaches the angle from the X -axis to perigee. For circular

TABLE 6-2. Computing the Classic Orbital Elements. For the right ascension of the ascending node, argument of perigee, and true anomaly, if the quantities in parentheses are positive, use the angle calculated. If the quantities are negative, the correct value is 360 deg minus the angle calculated.

Symbol	Name	Equation	Quadrant Check
a	semimajor axis	$a = -\mu/(2e) = (r_A + r_P)/2$	
e	eccentricity	$e = \mathbf{e} = 1 - (r_P/a) = (r_A/a) - 1$	
i	inclination	$i = \cos^{-1}(h_z/h)$	
Ω	right ascension of the ascending node	$\Omega = \cos^{-1}(n_x/n)$	$(n_y > 0)$
ω	argument of perigee	$\omega = \cos^{-1}[(n \cdot \mathbf{e})/(n \cdot \mathbf{e})]$	$(e_z > 0)$
ν	true anomaly	$\nu = \cos^{-1}[(\mathbf{e} \cdot \mathbf{r})/(\mathbf{e} \cdot \mathbf{r})]$	$(\mathbf{r} \cdot \mathbf{v} > 0)$
r_P	radius of perigee	$r_P = a(1 - e)$	
r_A	radius of apogee	$r_A = a(1 + e)$	
P	period	$P = 2\pi(a^3/\mu)^{1/2}$ $\approx 84.489 (a/R_E)^{3/2} \text{ min}$ $\approx 0.00016587 a^{3/2} \text{ min, } a \text{ in km}$	
ω_0	orbital frequency	$\omega_0 = (\mu/a^3)^{1/2}$ $\approx 631.348 \text{ } 16 a^{-3/2} \text{ rad/s, } a \text{ in km}$	

orbits ($e = 0$), a single angle, $u \equiv \omega + \nu$, can replace the argument of perigee and true anomaly. This angle is the **argument of latitude** and, when $e = 0$, equals the angle from the nodal vector to the satellite position vector. Finally, if the orbit is circular and equatorial, then a single angle, l , or **true longitude**, specifies the angle between the X -axis and the satellite position vector.

6.1.4 Satellite Ground Tracks

As defined in Chap. 5, a satellite's ground track is the trace of the points formed by the intersection of the satellite's position vector with the Earth's surface. In this section we will evaluate ground tracks using a flat map of the Earth. Chapters 5 and 7 give another approach of displaying them on a global representation.

Although ground tracks are generated from the orbital elements, we can gain insight by determining the orbital elements from a given ground track. Figure 6-4 shows ground tracks for satellites with different orbital altitudes and, therefore, different orbital periods. The time it takes Earth to rotate through the difference in longitude between two successive ascending nodes equals the orbit's period. For **direct** orbits, in which the satellite moves eastward, we measure the change positive to the east. For **retrograde** orbits, in which the satellite moves westward, positive is measured to the west.* With these definitions, the period, P , in minutes is

$$P = 4(360 \text{ deg} - \Delta L) \text{ direct orbit} \quad (6-10)$$

$$P = 4(\Delta L - 360 \text{ deg}) \text{ retrograde orbit}$$

* This convenient empirical definition does not apply for near polar orbits. More formally, a **prograde** or **direct** orbit has $i < 90$ deg. A **retrograde** orbit has $i > 90$ deg. A **polar orbit** has $i = 90$ deg.

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(For a more precise rotation rate for the Earth, use $3.988^\circ \cdot \text{min}/\text{deg}$ instead of 4.0 in these equations.) where ΔL is the change in longitude in degrees that the satellite goes through between successive ascending nodes. The difference in longitude between two successive ascending nodes for a direct orbit will always be less than 360 deg, and in fact will be negative for orbits at altitudes higher than geosynchronous altitude. For retrograde orbits, the difference in longitude between two successive ascending nodes (positive change is measured to the west) is always greater than 360 deg.

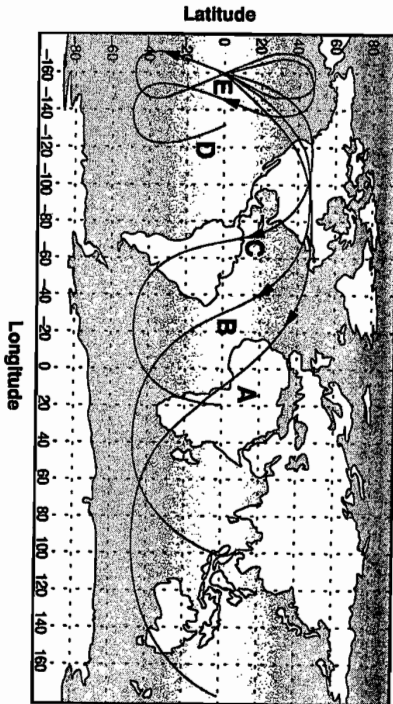


Fig. 6-4. Orbital Ground Tracks of Circular Orbits of Different Periods. (A) $\Delta L = 335^\circ$, $P = 100$ min; (B) $\Delta L = 280^\circ$, $P = 398$ min; (C) $\Delta L = 180^\circ$, $P = 718$ min; (D) $\Delta L = 28^\circ$, $P = 1,324$ min; and (E) $\Delta L = 0^\circ$, $P = 1,436$ min.

Once we know the period, we can determine the semimajor axis by using the equation for the period of an elliptical orbit:

$$a = [(P/2\pi)^2 \mu]^{1/3} \quad (6-11)$$

$$\cong 331,249 \text{ } 15 \text{ } P^{2/3} \text{ km}$$

where the period is in minutes.

Figure 6-4 shows one revolution each for the ground tracks of several orbits with an increasing semimajor axis. The period of a *geosynchronous* orbit, E , is $1,436$ min, matching the Earth's rotational motion.

We can determine the orbit's inclination by the ground track's maximum latitude. For direct orbits the inclination equals the ground track's maximum latitude, and for retrograde orbits the inclination equals 180 deg minus the ground track's maximum latitude.

The orbit is circular if a ground track is symmetrical about both the equator and a line of longitude extending downward from the ground track's maximum latitude. All the orbits in Fig. 6-4 are circular.

Figure 6-5 shows examples of ground tracks for the following typical orbits:

A: Shuttle parking orbit, $a = 6,700$ km, $e = 0$, $i = 28.4$ deg;

B: Low-altitude retrograde orbit, $a = 6,700$ km, $e = 0$, $i = 98.0$ deg;

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C: GPS orbit, $a = 26,600$ km, $e = 0$, $i = 55.0$ deg; and
D: Molniya orbits, $a = 26,600$ km, $e = 0.75$, $i = 63.4$ deg, $\omega = 270$ deg.

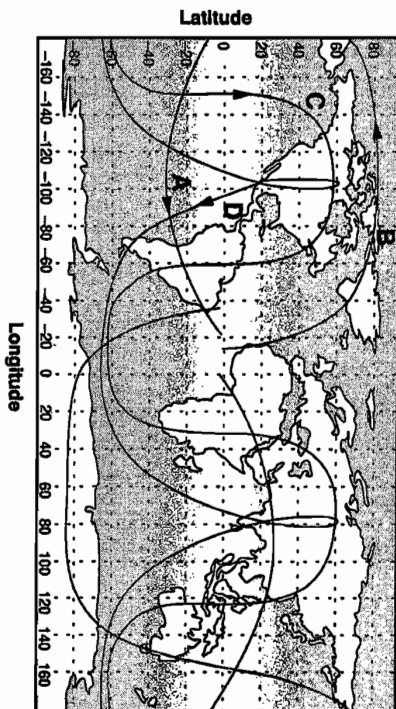


Fig. 6-5. Typical Ground Tracks. (A) Shuttle parking, (B) Low-altitude retrograde, (C) GPS, and (D) Molniya orbits. See text for orbital elements.

6.1.5 Time of Flight in an Elliptical Orbit

In analyzing Brabe's observational data, Kepler was able to solve the problem of relating position in the orbit to the elapsed time, $t - t_0$, or conversely, how long it takes to go from one point to another in an orbit. To solve this, Kepler introduced the quantity M , called the *mean anomaly*, which is the fraction of an orbit period which has elapsed since perigee, expressed as an angle. The mean anomaly equals the true anomaly for a circular orbit. By definition,

$$M - M_0 = n(t - t_0) \quad (6-12)$$

where M_0 is the mean anomaly at time t_0 and n is the *mean motion*, or average angular velocity, determined from the semimajor axis of the orbit:

$$n \cong (\mu/a^3)^{1/2} \quad (6-13)$$

$$\cong 36,173.585 \text{ } a^{-3/2} \text{ deg/s}$$

$$\cong 8,681,660.4 \text{ } a^{-3/2} \text{ rev/day}$$

$$\cong 3.125 \text{ } 297 \text{ } 7 \times 10^8 \text{ } a^{-3/2} \text{ deg/day}$$

where a is in km.

This solution will give the average position and velocity, but satellite orbits are elliptical, with a radius constantly varying in orbit. Because the satellite's velocity depends on this varying radius, it changes as well. To resolve this problem we define an intermediate variable called *eccentric anomaly*, E , for elliptical orbits. Table 6-3 lists the equations necessary to relate time of flight to orbital position.

TABLE 6-3. Time of Flight in an Elliptic Orbit. All angular quantities are in radians.

Variable	Name	Equation	
n	mean motion	$n = (\mu/a^3)^{1/2}$ $= 631.348 \text{ } 16 \text{ } a^{-3/2} \text{ rad/s}$	(a in km)
E	eccentric anomaly	$\cos E = (e + \cos v)/(1 + e \cos v)$	
M	mean anomaly	$M = E - e \sin E$ $M = M_0 + n(t - t_0)$	(M in rad) (M in rad)
$t - t_0$	time of flight	$t - t_0 = (M - M_0)/n$	($t - t_0$ in sec)
v	true anomaly	$v = M + 2e \sin M + 1.25e^2 \sin(2M)$ (approx.)	

As an example, we find the time it takes a satellite to go from perigee to an angle 90 deg from perigee, for an orbit with a semimajor axis of 7,000 km and an eccentricity of 0.1. For this example

$$\begin{aligned} \gamma_0 = E_0 = M_0 &= 0.0 \text{ rad} & t_0 &= 0.0 \text{ s} \\ \nu &= 1.5708 \text{ rad} & E &= 1.4706 \text{ rad} \\ M &= 1.3711 \text{ rad} & n &= 0.00108 \text{ rad/s} \\ t &= 1,271.88 \text{ s} \end{aligned}$$

Finding the position in an orbit after a specified period is more complex. For this problem, we calculate the mean anomaly, M , using time of flight and the mean motion using Eq. (6-12). Next, we determine the true anomaly, v , using the series expansion shown in Table 6-3, a good approximation for small eccentricity (the error is of the order e^3). If we need greater accuracy, we must solve the equation in Table 6-3 relating mean anomaly to eccentric anomaly. Because this is a transcendental function, we must use an iterative solution to find the eccentric anomaly, after which we can calculate the true anomaly directly.

6.1.6 Orbit Determination

Up to this point, we have assumed that we know both the position and velocity of the satellite in inertial space or the classical orbital elements. But we often cannot directly observe the satellite's inertial position and velocity. Instead, we commonly receive data from radar, telemetry, optics, or GPS. Radar and telemetry data consists of range, azimuth, elevation, and possibly the rates of change of one or more of these quantities, relative to a site attached to the rotating Earth. GPS receivers give GCI latitude, longitude, and altitude. Optical data consists of right ascension and declination relative to the celestial sphere. In any case, we must combine and convert this data to inertial position and velocity before determining the orbital elements. Bate, Mueller, and White [1971] and Escobal [1965] cover methods for combining data, so I will not cover them here.

The type of data we use for orbit determination depends on the orbit selected, accuracy requirements, and weight restrictions on the spacecraft. Because radar and optical systems collect data passively, they require no additional spacecraft weight, but they are also the least accurate methods of orbit determination. Conversely, GPS

data is more accurate but also requires a receiver and processor, which add weight. We can also use GPS for semiautonomous orbit determination because it requires no ground support. An alternative method for fully autonomous navigation is given by Tai and Noerdlinger [1989] (See Sec. 11.2.)

6.2 Orbit Perturbations

The Keplerian orbit discussed above provides an excellent reference, but other forces act on the satellite to perturb it away from the nominal orbit. We can classify these *perturbations*, or variations in the orbital elements, based on how they affect the Keplerian elements.

Figure 6-6 illustrates a typical variation in one of the orbital elements because of a perturbing force. *Secular variations* represent a linear variation in the element. *Short-period variations* are periodic in the element with a period less than or equal to the orbital period. *Long-period variations* have a period greater than the orbital period. Because secular variations have long-term effects on orbit prediction (the orbital elements affected continue to increase or decrease), I will discuss them in detail. If the satellite mission demands that we precisely determine the orbit, we must include the periodic variations as well. Baitin [1999], Danby [1962], and Escobal [1965] describe methods of determining and predicting orbits for perturbed Keplerian motion.

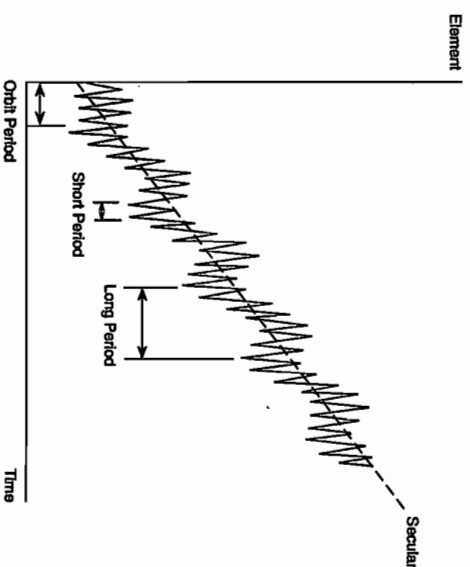


Fig. 6-6. Secular and Periodic Variations of an Orbital Element. Secular variations represent linear variations in the element, short-period variations have a period less than the orbital period, and long-period variations have a period longer than the orbital period.

When we consider perturbing forces, the classical orbital elements vary with time. To predict the orbit we must determine this time variation using techniques of either

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special or general perturbations. *Special perturbations* employ direct numerical integration of the equations of motion. Most common is *Cowell's method*, in which the accelerations are integrated directly to obtain velocity and again to obtain position.

General perturbations analytically solve some aspects of the motion of a satellite subjected to perturbing forces. For example, the polar equation of a conic applies to the two-body equations of motion. Unfortunately, most perturbing forces don't yield to a direct analytic solution but to series expansions and approximations. Because the orbital elements are nearly constant, general perturbation techniques usually solve directly for the orbital elements rather than the inertial position and velocity. The orbital elements are more difficult to describe mathematically and approximate, but they allow us to better understand how perturbations affect a large class of orbits. We can also obtain solutions much faster than with special perturbations.

The primary forces which perturb a satellite orbit arise from third bodies such as the Sun and the Moon, the nonspherical mass distribution of the Earth, atmospheric drag, and solar radiation pressure. We describe each of these below.

6.2.1 Third-Body Perturbations

The gravitational forces of the Sun and the Moon cause periodic variations in all of the orbital elements, but only the right ascension of the ascending node, argument of perigee, and mean anomaly experience secular variations. These secular variations arise from a gyroscopic precession of the orbit about the ecliptic pole. The secular variation in mean anomaly is much smaller than the mean motion and has little effect on the orbit; however, the secular variations in right ascension of the ascending node and argument of perigee are important, especially for high-altitude orbits.

For nearly circular orbits, e^2 is almost zero and the resulting error is of the order e^2 . In this case, the equations for the secular rates of change resulting from the Sun and Moon are:

- Right ascension of the ascending node:

$$\dot{\Omega}_{MOON} = -0.00338 (\cos i) / n \quad (6-14)$$

$$\dot{\Omega}_{SUN} = -0.00154 (\cos i) / n \quad (6-15)$$

- Argument of perigee:

$$\dot{\omega}_{MOON} = 0.00169 (4 - 5 \sin^2 i) / n \quad (6-16)$$

$$\dot{\omega}_{SUN} = 0.00077 (4 - 5 \sin^2 i) / n \quad (6-17)$$

where i is the orbital inclination, n is the number of orbit revolutions per day, and $\dot{\Omega}$ and $\dot{\omega}$ are in deg/day. These equations are only approximate; they neglect the variation caused by the changing orientation of the orbital plane with respect to both the Moon's orbital plane and the ecliptic plane.

6.2.2 Perturbations Because of a Nonspherical Earth

When developing the two-body equations of motion, we assumed the Earth has a spherically symmetric mass distribution. In fact, the Earth has a bulge at the equator,

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a slight pear shape, and flattening at the poles. We can find a satellite's acceleration by taking the gradient of the gravitational potential function, Φ . One widely used form of the geopotential function is:

$$\Phi = (\mu/r) \left[1 - \sum_{n=2}^{\infty} J_n (R_E/r)^n P_n(\sin L) \right] \quad (6-18)$$

where $\mu = GM$ is Earth's gravitational constant, R_E is Earth's equatorial radius, P_n are Legendre polynomials, L is geocentric latitude, and J_n are dimensionless geopotential coefficients of which the first three are:

$$\begin{aligned} J_2 &= 0.00108263 \\ J_3 &= -0.00000254 \\ J_4 &= -0.00000161 \end{aligned}$$

This form of the geopotential function depends on latitude, and we call the geopotential coefficients, J_n , *zonal coefficients*. Other, more general expressions for the geopotential include sectoral and tesseral terms in the expansion. The *sectoral terms* divide the Earth into slices and depend only on longitude. The *tesseral terms* in the expansion represent sections that depend on longitude and latitude. They divide the Earth into a checkerboard pattern of regions that alternately add to and subtract from the two-body potential. A *geopotential model* consists of a matrix of coefficients in the spherical harmonic expansion. The widely used Goddard Earth Model 10B, or GEM10B, is called a "21 × 21 model" because it consists of a 21 × 21 matrix of coefficients. In order to achieve high accuracy mapping of the ocean surface and wave properties, the TOPEX mission required creating a 100 × 100 geopotential model.

The potential generated by the nonspherical Earth causes periodic variations in all of the orbital elements. The dominant effects, however, are secular variations in right ascension of the ascending node and argument of perigee because of the Earth's oblateness, represented by the J_2 term in the geopotential expansion. The rates of change of $\dot{\Omega}$ and $\dot{\omega}$ due to J_2 are

$$\begin{aligned} \dot{\Omega}_2 &= -1.5n J_2 (R_E/a)^2 (\cos i) (1 - e^2)^{-2} \\ &\approx -2.06474 \times 10^{-4} a^{-7/2} (\cos i) (1 - e^2)^{-2} \end{aligned} \quad (6-19)$$

$$\begin{aligned} \dot{\omega}_2 &= 0.75n J_2 (R_E/a)^2 (4 - 5 \sin^2 i) (1 - e^2)^{-2} \\ &\approx 1.03237 \times 10^{-4} a^{-7/2} (4 - 5 \sin^2 i) (1 - e^2)^{-2} \end{aligned} \quad (6-20)$$

where n is mean motion in deg/day, R_E is Earth's equatorial radius, a is semimajor axis in km, e is eccentricity, i is inclination, and $\dot{\Omega}$ and $\dot{\omega}$ are in deg/day. Table 6-4 compares the rates of change of right ascension of the ascending node and argument of perigee resulting from the Earth's oblateness, the Sun, and the Moon. For satellites in GEO and below, the J_2 perturbations dominate; for satellites above GEO the Sun and Moon perturbations dominate.

Molniya orbits are highly eccentric ($e \approx 0.75$) with approximately 12 hour periods (2 revolutions/day). Orbit designers choose the orbital inclination so the rate of change of perigee, Eq. (6-20), is zero. This condition occurs at inclinations of 63.4 deg and 116.6 deg. For these orbits we typically place perigee in the Southern Hemisphere, so

TABLE 6-4. Secular Variations in Flight Ascension of the Ascending Node and Argument of Perigee. Note that these secular variations form the basis for Sun-synchronous and Molniya orbits. For Sun-synchronous orbits the nodal precession rate is set to 0.986 deg/day to match the general motion of the Sun.

Orbit	Effect of J_2 (Eqs. 6-19, 6-20) (deg/day)	Effect of the Moon (Eqs. 6-14, 6-16) (deg/day)	Effect of the Sun (Eqs. 6-15, 6-17) (deg/day)
Shuttle	$a = 6,700$ km, $e = 0.0$, $i = 28$ deg		
$\dot{\Omega}$	-7.35	-0.000 19	-0.000 08
$\dot{\omega}$	12.05	0.000 31	0.000 14
Sun-Synchronous	$a = 6,728$ km, $e \equiv 0.0$, $i = 96.85$ deg		
$\dot{\Omega}$	0.986	0.000 03	0.000 01
$\dot{\omega}$	-4.890	-0.000 10	-0.000 05
GPS	$a = 26,600$ km, $e = 0.0$, $i = 60.0$ deg		
$\dot{\Omega}$	-0.033	-0.000 85	-0.000 38
$\dot{\omega}$	0.008	0.000 21	0.000 10
Molniya	$a = 26,600$ km, $e = 0.75$, $i = 63.4$ deg		
$\dot{\Omega}$	-0.30	-0.000 76	-0.000 34
$\dot{\omega}$	0.00	0.000 00	0.000 00
Geosynchronous	$a = 42,160$ km, $e = 0$, $i = 0$ deg		
$\dot{\Omega}$	-0.013	-0.003 38	-0.001 54
$\dot{\omega}$	0.025	0.006 76	0.003 07

the satellite remains above the Northern Hemisphere near apogee for approximately 11 hours/orbit. Mission planners choose perigee altitude to meet the satellite's mission constraints. Typical perigee altitudes vary from 200 to 1,000 km. We can calculate the eccentricity and apogee altitude using the semimajor axis, perigee altitude, and equations from Table 6-2.

In a *Sun-synchronous orbit*, the satellite orbital plane remains approximately fixed with respect to the Sun. We do this by matching the secular variation in the right ascension of the ascending node (Eq. 6-19) to the Earth's rotation rate around the Sun. A nodal precession rate of 0.986 deg/day will match the Earth's average rotation rate about the Sun. Because this rotation is positive, Sun-synchronous orbits must be retrograde. For a given semimajor axis, a , and eccentricity, we can use Eq. (6-19) to find the inclination for the orbit to be Sun-synchronous.

6.2.3 Perturbations From Atmospheric Drag

The principal nongravitational force acting on satellites in low-Earth orbit is atmospheric drag. Drag acts in the opposite direction of the velocity vector and removes energy from the orbit. This energy reduction causes the orbit to get smaller,

leading to further increases in drag. Eventually, the altitude of the orbit becomes so small that the satellite reenters the atmosphere.

The equation for acceleration due to drag on a satellite is:

$$a_D = -(1/2)\rho(C_D A/m)V^2 \quad (6-21)$$

where ρ is atmospheric density, A is the satellite's cross-sectional area, m is the satellite's mass, V is the satellite's velocity with respect to the atmosphere, and C_D is the drag coefficient ≈ 2.2 (See Table 8-3 in Sec. 8.1.3 for an extended discussion of C_D).

We can approximate the changes in semimajor axis and eccentricity per revolution, and the lifetime of a satellite, using the following equations:

$$\Delta a_{rev} = -2\pi(C_D A/m)a^2 \rho_p \exp(-c) [l_0 + 2e l_1] \quad (6-22)$$

$$\Delta e_{rev} = -2\pi(C_D A/m)a \rho_p \exp(-c) [l_1 + e(l_0 + l_2)/2] \quad (6-23)$$

where ρ_p is atmospheric density at perigee, $c \equiv ae/H$, H is density scale height (see column 25, Inside Rear Cover), and l_i are Modified Bessel Functions* of order i and argument c . We model the term $m/(C_D A)$, or *ballistic coefficient*, as a constant for most satellites, although it can vary by a factor of 10 depending on the satellite's orientation (see Table 8-3).

For near circular orbits, we can use the above equations to derive the much simpler expressions:

$$\Delta a_{rev} = -2\pi(C_D A/m)a^2 \quad (6-24)$$

$$\Delta P_{rev} = -6\pi^2(C_D A/m)a^2/V \quad (6-25)$$

$$\Delta V_{rev} = \pi(C_D A/m)aV \quad (6-26)$$

$$\Delta e_{rev} = 0 \quad (6-27)$$

where P is orbital period and V is satellite velocity.

A rough estimate of the satellite's lifetime, L , due to drag is

$$L \approx -H/\Delta a_{rev} \quad (6-28)$$

where, as above, H is atmospheric density scale height given in column 25 of the Earth Satellite Parameter tables in the back of this book. We can obtain a substantially more accurate estimate (although still very approximate) by integrating Eq. (6-24), taking into account the changes in atmospheric density with both altitude and solar activity level. We did this for representative values of the ballistic coefficient in Fig. 8-4 in Sec. 8.1.

6.2.4 Perturbations from Solar Radiation

Solar radiation pressure causes periodic variations in all of the orbital elements. Its effect is strongest for satellites with low ballistic coefficients, that is, light vehicles with large frontal areas such as Echo. The magnitude of the acceleration, a_R , in m/s^2 arising from solar radiation pressure is

$$a_R \approx -4.5 \times 10^{-6} (1 + r) A/m \quad (6-29)$$

where A is the satellite cross-sectional area exposed to the Sun in m^2 , m is the satellite mass in kg, and r is a reflection factor. ($r = 0$ for absorption; $r = 1$ for specular

* Values for l_i can be found in many standard mathematical tables.

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reflection at normal incidence; and $r \approx 0.4$ for diffuse reflection.) Below 800 km altitude, acceleration from drag is greater than that from solar radiation pressure; above 800 km, acceleration from solar radiation pressure is greater.

6.3 Orbit Maneuvering

At some point during the lifetime of most satellites, we must change one or more of the orbital elements. For example, we may need to transfer it from an initial parking orbit to the final mission orbit, rendezvous with or intercept another satellite, or correct the orbital elements to adjust for the perturbations discussed in the previous section. Most frequently, we must change the orbital altitude, plane, or both. To change the orbit of a satellite, we have to change the satellite's velocity vector in magnitude or direction using a thruster. Most propulsion systems operate for only a short time compared to the orbital period, so we can treat the maneuver as an impulsive change in the velocity while the position remains fixed. For this reason, any maneuver changing the orbit of a satellite must occur at a point where the old orbit intersects the new orbit. If the two orbits do not intersect, we must use an intermediate orbit that intersects both. In this case, the total maneuver requires at least two propulsive burns.

In general, the change in the velocity vector to go from one orbit to another is given by

$$\Delta \mathbf{V} = \mathbf{V}_{\text{NEED}} - \mathbf{V}_{\text{CURRENT}} \quad (6-30)$$

We can find the current and needed velocity vectors from the orbital elements, keeping in mind that the position vector does not change significantly during impulsive burns.

6.3.1 Coplanar Orbit Transfers

The most common type of in-plane maneuver changes the size and energy of the orbit, usually from a low-altitude parking orbit to a higher-altitude mission orbit such as a geosynchronous orbit. Because the initial and final orbits do not intersect (see Fig. 6-7), the maneuver requires a transfer orbit. Figure 6-7 represents a Hohmann* Transfer Orbit. In this case, the transfer orbit's ellipse is tangent to both the initial and final circular orbits at the transfer orbit's perigee and apogee, respectively. The orbits are tangential, so the velocity vectors are collinear at the intersection points, and the Hohmann Transfer represents the most fuel-efficient transfer between two circular, coplanar orbits. When transferring from a smaller orbit to a larger orbit, the propulsion system must apply velocity change in the direction of motion; when transferring from a larger orbit to a smaller, the velocity change is opposite to the direction of motion.

The total velocity change required for the transfer is the sum of the velocity changes at perigee and apogee of the transfer ellipse. Because the velocity vectors are collinear at these points, the velocity changes are just the differences in magnitudes of the velocities in each orbit. We can find these differences from the energy equation, if we know the size of each orbit. If we know the initial and final orbits (r_A and r_B), we calculate the semimajor axis of the transfer ellipse, a_{tr} , and the total velocity change

*Walter Hohmann, a German engineer and architect, wrote *The Attainability of Celestial Bodies* [1925], consisting of a mathematical discussion of the conditions for leaving and returning to Earth.

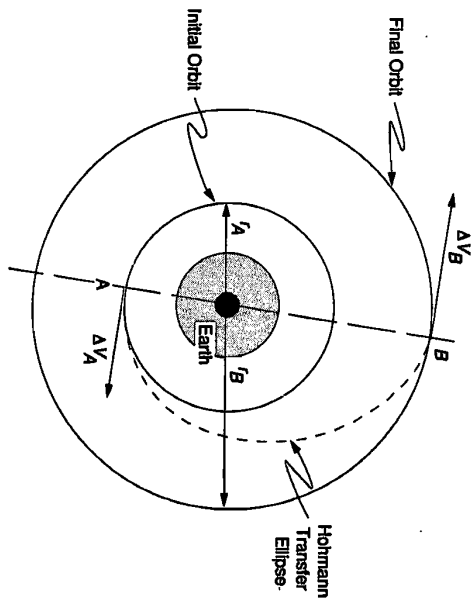


Fig. 6-7. Hohmann Transfer. The Hohmann Transfer ellipse provides orbit transfer between two circular, co-planar orbits.

(the sum of the velocity changes required at points A and B) using the following algorithm. An example transferring from an initial circular orbit of 6,567 km to a final circular orbit of 42,160 km illustrates this technique.

STEP	EQUATIONS	EXAMPLE
1.	$a_{tr} = (r_A + r_B)/2$	$= 24,364 \text{ km}$
2.	$V_{tA} = (\mu/r_A)^{1/2} = 631.3481(r_A)^{-1/2}$	$= 7.79 \text{ km/s}$
3.	$V_{tB} = (\mu/r_B)^{1/2} = 631.3481(r_B)^{-1/2}$	$= 3.08 \text{ km/s}$
4.	$V_{tA} = [\mu(2/r_A - 1/a_{tr})]^{1/2} = 631.3481[(2/r_A - 1/a_{tr})]^{1/2}$	$= 10.25 \text{ km/s}$
5.	$V_{tB} = [\mu(2/r_B - 1/a_{tr})]^{1/2} = 631.3481[(2/r_B - 1/a_{tr})]^{1/2}$	$= 1.59 \text{ km/s}$
6.	$\Delta V_A = V_{tA} - V_{tA} $	$= 2.46 \text{ km/s}$
7.	$\Delta V_B = V_{tB} - V_{tB} $	$= 1.49 \text{ km/s}$
8.	$\Delta V_{\text{TOTAL}} = \Delta V_A + \Delta V_B$	$= 3.95 \text{ km/s}$
9.	Time of transfer $= P/2$	$= 5 \text{ hr } 15 \text{ min}$

Alternatively, we can write the total ΔV required for a two-burn transfer between circular orbits at radii r_A and r_B :

$$\Delta V_{\text{TOTAL}} \equiv \Delta V_A + \Delta V_B \quad (6-31)$$

$$= \sqrt{\mu} \left[\left| \left(\frac{2}{r_A} - \frac{1}{a_{\text{tr}}} \right)^2 - \left(\frac{1}{r_A} \right)^2 \right| + \left| \left(\frac{2}{r_B} - \frac{1}{a_{\text{tr}}} \right)^2 - \left(\frac{1}{r_B} \right)^2 \right| \right] \quad (6-32)$$

where $\sqrt{\mu} = 631.3481$ when ΔV is in km/s and all of the semimajor axes are in km. As in step 1 above, $a_{\text{tr}} = (r_A + r_B)/2$.

The above expression applies to any coplanar Hohmann transfer. In the case of small transfers (that is, r_A close to r_B), we can approximate this conveniently in two forms:

$$\Delta V \approx V_A - V_B \quad (6-33)$$

$$\Delta V \approx 0.5 (\Delta r / r) V_{A/B} \quad (6-34)$$

where

$$\Delta r \equiv r_B - r_A \quad (6-35)$$

and

$$r \approx r_A \approx r_B \quad V_{A/B} \approx V_A \approx V_B \quad (6-36)$$

To make the orbit change, we divide the ΔV into two small burns are of approximately equal magnitude.

The result in Eq. (6-33) is more unusual than it might seem at first. Assume that a satellite is in a circular orbit with velocity V_A . In two burns we increase the velocity by an amount ΔV . The result is that the satellite is higher and traveling slower than originally by the amount ΔV . We can best clarify this result by an example. Consider a satellite initially in a circular orbit at 400 km such that $r_A = 6,778$ km and $V_A = 7,700$ m/s. We will apply a total ΔV of 20 m/s ($\approx 0.26\%$ of V_A) in two burns of 10 m/s each. From Eq. (6-34) the total Δr will be 0.52% of 6,778 km or 35 km. Thus, the final orbit will be circular at an altitude of 6,813 km. Immediately following the first burn of 10 m/s the spacecraft will be at perigee of the transfer orbit with a velocity of 7,710 m/s. When the spacecraft reaches apogee at 6,813 km it will have slowed according to Kepler's second law by 0.52% to 7,670 m/s. We then apply the second burn of 10 m/s to circularize the orbit at 7,680 m/s which is 20 m/s slower than its original velocity. We have added energy to the spacecraft which has raised the orbit and resulted in a lower kinetic energy but sufficiently more potential energy to make up for both the reduced speed and the added ΔV .

Sometimes, we may need to transfer a satellite between orbits in less time than that required to complete the Hohmann transfer. Figure 6-8 shows a faster transfer called the *one-tangent burn*. In this instance the transfer orbit is tangential to the initial orbit. It intersects the final orbit at an angle equal to the flight-path angle of the transfer orbit at the point of intersection. An infinite number of transfer orbits are tangential to the initial orbit and intersect the final orbit at some angle. Thus, we may choose the transfer orbit by specifying the size of the transfer orbit, the angular change of the transfer, or the time required to complete the transfer. We can then define the transfer orbit and calculate the required velocities.

For example, we may specify the size of the transfer orbit, choosing any semimajor axis that is greater than the semimajor axis of the Hohmann transfer ellipse. Once we know the semimajor axis of the ellipse (a_{tr}), we can calculate the eccentricity, angular distance traveled in the transfer, the velocity change required for the transfer, and the time required to complete the transfer using the equations in Table 6-5.

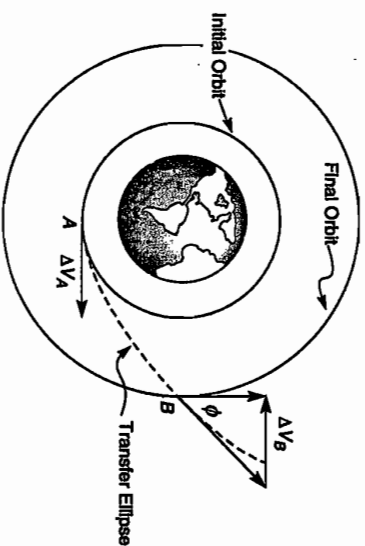


Fig. 6-8. Transfer Orbit Using One-Tangent Burn between Two Circular, Coplanar Orbits.

TABLE 6-5. Computations for One-Tangent Burn Orbit Transfer. See Vallado [1997] for examples. Given: r_A , r_B , and a_{tr} .

Quantity	Equation
eccentricity	$e = 1 - r_A / a_{\text{tr}}$
true anomaly at second burn	$v = \cos^{-1}[(a_{\text{tr}}(1 - e^2) / r_B - 1) / e]$
flight-path angle at second burn	$\phi = \tan^{-1}[e \sin v / (1 + e \cos v)]$
Initial velocity	$V_A = 631.3481 / r_A^{1/2}$
velocity on transfer orbit at initial orbit	$V_{Aa} = 631.3481 [2 / r_A - 1 / a_{\text{tr}}]^{1/2}$
Initial velocity change	$\Delta V_A = V_{Aa} - V_A $
final velocity	$V_B = 631.3481 / r_B^{1/2}$
velocity on transfer orbit at final orbit	$V_{Ba} = 631.3481 [2 / r_B - 1 / a_{\text{tr}}]^{1/2}$
final velocity change	$\Delta V_B = [V_B^2 + V_{Ba}^2 - 2 V_B V_{Ba} \cos \phi]^{1/2}$
total velocity change	$\Delta V_T = \Delta V_A + \Delta V_B$
eccentric anomaly at B	$E = \tan^{-1}[(1 - e^2)^{1/2} \sin v / (e + \cos v)]$
time of flight	$\text{TOF} = 0.001583913 a_{\text{tr}}^{3/2} [E - e \sin E]$, E in rads

Table 6-6 compares the total velocity change required and time-of-flight for a Hohmann transfer and a one-tangent burn transfer from a low altitude parking orbit to geosynchronous orbit.

Another option for changing the size of the orbit is to use a constant low-thrust burn, which results in a *spiral transfer*. We can approximate the velocity change for this type of orbit transfer by

$$\Delta V = |V_2 - V_1| \quad (6-37)$$

where the velocities are the circular velocities of the two orbits. Following the

TABLE 6-6. Comparison of Coplanar Orbit Transfers from LEO to Geosynchronous Orbit.

Variable	Hohmann Transfer	One-Tangent-Burn
r_A	6,570 km	6,570 km
r_B	42,200 km	42,200 km
a_B	24,385 km	28,633 km
ΔV_T	3.935 km/s	4.699 km/s
TOF	5.256 hr	3.457 hr

previous example, the total velocity change required to go from low-Earth orbit to geosynchronous is 4.71 km/s using a spiral transfer. We obtain this by subtracting the results of step 3 from the results of step 2 in the above example.

6.3.2 Orbit Plane Changes

To change the orientation of the satellite's orbital plane, typically the inclination, we must change the direction of the velocity vector. This maneuver requires a component of ΔV to be perpendicular to the orbital plane and, therefore, perpendicular to the initial velocity vector. If the size of the orbit remains constant, we call the maneuver a *simple plane change* (Fig. 6-9A). We can find the required change in velocity by using the law of cosines. For the case in which V_i is equal to V_f , this expression reduces to

$$\Delta V = 2V_i \sin(\theta/2) \quad (6-38)$$

where V_i is the velocity before and after the burn, and θ is angle change required.



Fig. 6-9. Vector Representation of Simple and Combined Changes in Orbital Plane. For the simple plane change, the magnitude of initial and final velocities are equal.

For example, the change in velocity required to transfer from a low-altitude ($h = 185$ km) inclined ($i = 28$ deg) orbit to an equatorial orbit ($i = 0$) at the same altitude is:

$$r = 6,563 \text{ km} \quad V_i = 7.79 \text{ km/s} \quad \Delta V = 3.77 \text{ km/s}$$

From Eq. (6-38) we see that if the angular change equals 60 deg, the required change in velocity equals the current velocity. Plane changes are very expensive in terms of the required velocity change and resulting fuel consumption. To minimize this, we should change the plane at a point where the velocity of the satellite is a minimum: at apogee for an elliptical orbit. In some cases, it may even be cheaper to boost the satellite into a higher orbit, change the orbital plane at apogee, and return the satellite to its original orbit.

Typically, orbital transfers require changes in both the size and the plane of the orbit, such as transferring from an inclined parking orbit at low altitude to a zero-inclination orbit at geosynchronous altitude. We do this transfer in two steps: a Hohmann transfer to change the size of the orbit and a simple plane change to make the orbit equatorial. A more efficient method (less total change in velocity) would be to combine the plane change with the tangential burn at apogee of the transfer orbit (Fig. 6-9B). As we must change both the magnitude and direction of the velocity vector, we can find the required change in velocity using the law of cosines:

$$\Delta V = (V_i^2 + V_f^2 - 2V_i V_f \cos \theta)^{1/2} \quad (6-39)$$

where V_i is the initial velocity, V_f is the final velocity, and θ is the angle change required.

For example, we find the total change in velocity to transfer from a Shuttle parking orbit to a geosynchronous equatorial orbit as follows:

$$\begin{aligned} r_i &= 6,563 \text{ km} & r_f &= 42,159 \text{ km} \\ i_i &= 28 \text{ deg} & i_f &= 0 \text{ deg} \\ V_i &= 7.79 \text{ km/s} & V_f &= 3.08 \text{ km/s} \\ \Delta V_A &= 2.46 \text{ km/s} & \Delta V_B &= 1.83 \text{ km/s} \\ \Delta V_{\text{TOTAL}} &= 4.29 \text{ km/s} \end{aligned}$$

Completing a Hohmann transfer followed by a simple plane change would require a velocity change of 5.44 km/s, so the Hohmann transfer with a combined plane change at apogee of the transfer orbit represents a savings of 1.15 km/s. As we see from Eq. (6-39), a small plane change ($\theta \geq 0$) can be combined with an energy change for almost no cost in ΔV or propellant. Consequently, in practice, we do geosynchronous transfer with a small plane change at perigee and most of the plane change at apogee.

Another option is to complete the maneuver using three burns. The first burn is a coplanar maneuver placing the satellite into a transfer orbit with an apogee much higher than the final orbit. When the satellite reaches apogee of the transfer orbit, it does a combined plane change maneuver. This places the satellite in a second transfer orbit which is coplanar with the final orbit and has a perigee altitude equal to the altitude of the final orbit. Finally, when the satellite reaches perigee of the second transfer orbit, another coplanar maneuver places the satellite into the final orbit. This three-burn maneuver may save fuel, but the fuel savings comes at the expense of the total time required to complete the maneuver.

6.3.3 Orbit Rendezvous

Orbital transfer becomes more complicated when the objective is to rendezvous with or intercept another object in space: both the interceptor and target must arrive at the rendezvous point at the same time. This precise timing demands a phasing orbit to accomplish the maneuver. A *phasing orbit* is any orbit which results in the interceptor achieving the desired geometry relative to the target to initiate a Hohmann transfer. If the initial and final orbits are circular, coplanar, and of different sizes, then the phasing orbit is simply the initial interceptor orbit (Fig. 6-10). The interceptor remains in the initial orbit until the relative motion between the interceptor and target results in the

desired geometry. At that point, we inject the interceptor into a Hohmann transfer orbit. The equation to solve for the wait time in the initial orbit is:

$$\text{Wait Time} = (\phi_i - \phi_f + 2\pi) / (\omega_{in} - \omega_{gr}) \quad (6-40)$$

where ϕ_i is the phase angle (angular separation of target and interceptor) needed for rendezvous, ϕ_f is the initial phase angle, k is the number of rendezvous opportunities, (for the first opportunity, $k=0$), ω_{in} is the angular velocity of the interceptor, and ω_{gr} is the angular velocity of the target. We calculate the lead angle, α_L , by multiplying ω_{gr} by the time of flight for the Hohmann transfer and ϕ_f is 180 deg minus α_L .

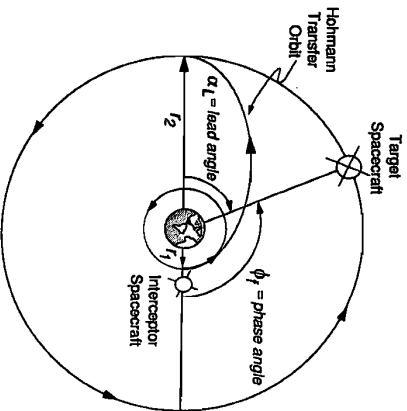


Fig. 6-10. Geometry Depicting Rendezvous Between Two Circular, Coplanar Orbits. The phase angle is the angular separation between the target and interceptor at the start of the rendezvous and the lead angle is the distance the target travels from the start until rendezvous occurs.

The total time to rendezvous equals the wait time from Eq. (6-40) plus the time of flight of the Hohmann transfer orbit.

The denominator in Eq. (6-40) represents the relative motion between the interceptor and target. As the size of the interceptor orbit approaches the size of the target orbit, the relative motion approaches zero, and the wait time approaches infinity. If the two orbits are exactly the same, then the interceptor must enter a new phasing orbit to rendezvous with the target (Fig. 6-11). For this situation, the rendezvous occurs at the point where the interceptor enters the phasing orbit. The period of the phasing orbit equals the time it takes the target to get to the rendezvous point. Once we know the period, we can calculate the semimajor axis. The two orbits are tangential at their point of intersection, so the velocity change is the difference in magnitudes of the two velocities at the point of intersection of the two orbits. Because we know the size of the orbits, and therefore, the energies, we can use the energy Eq. (6-4) to solve for the current and needed velocities.

Frequently operators must adjust the relative phasing for satellites in circular orbits. They accomplish this by making the satellite drift relative to its initial position. The drift rate in deg/orbit is given by

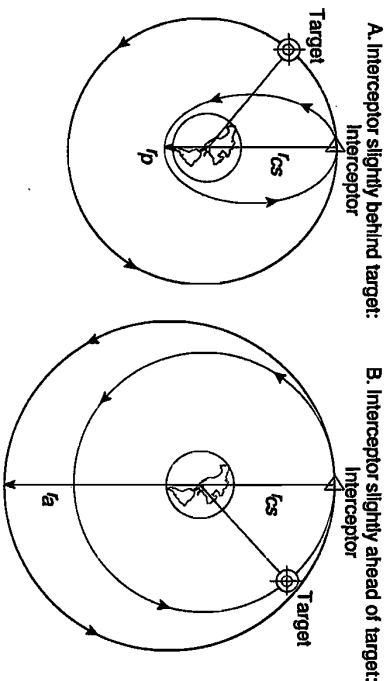


Fig. 6-11. Rendezvous from Same Orbit Showing the Target both Leading and Trailing the Interceptor.

$$\text{drift rate} = 1,080 \Delta V / V \quad (6-41)$$

where V is the nominal orbital velocity and ΔV is the velocity change required to start or stop the drift.

The techniques described above move the target vehicle close to the interceptor. Once the two vehicles are close to each other they begin proximity operations by solving a set of relative motion equations to achieve the final rendezvous. Vallado [1997] contains an excellent discussion of the solution to the nearby relative motion problem, as addressed by the *Clohesy-Wiltshire* or *Hill's equations* of relative motion.

6.4 Launch Windows

Similar to the rendezvous problem is the launch-window problem, or determining the appropriate time to launch from the surface of the Earth into the desired orbital plane. Because the orbital plane is fixed in inertial space, the launch window is the time when the launch site on the surface of the Earth rotates through the orbital plane. As Fig. 6-12 shows, the launch time depends on the launch site's latitude and longitude and the satellite orbit's inclination and right ascension of the ascending node.

For a launch window to exist, the launch site must pass through the orbital plane. This requirement places restrictions on the orbital inclinations, i , possible from a given launch latitude, L :

- No launch windows exist if $L > i$ for direct orbit or $L > 180 \text{ deg} - i$ for retrograde orbits.
- One launch window exists if $L = i$ or $L = 180 \text{ deg} - i$.
- Two launch windows exist if $L < i$ or $L < 180 \text{ deg} - i$.

The *launch azimuth*, β , is the angle measured clockwise from north to the velocity vector. If a launch window exists, then the launch azimuth required to achieve an inclination, i , from a given launch latitude, L , is given by:

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Fig. 6-12 Launch Window Geometry for Launches near the Ascending Node (1) and Descending Node (2). The angles shown are the orbital inclination (i), launch site latitude (L), and launch azimuth (β).

$$\beta = \beta_i \pm \gamma \approx \beta_i \quad (6-42a)$$

where

$$\sin \beta_i = \cos i / \cos L \quad (6-42b)$$

and

$$\tan \gamma = \frac{V_L \cos \beta_i}{V_0 - V_{eq} \cos i} \approx \left(\frac{V_L}{V_0} \right) \cos \beta_i \quad (6-42c)$$

where V_L is the inertial velocity of the launch site given by Eq. (6-46) below, $V_{eq} = 464.5$ m/s is the velocity of Earth's rotation at the equator, and $V_0 \approx 7.8$ km/s is the velocity of the satellite immediately after launch. β_i is the inertial launch azimuth and γ is a small correction to account for the velocity contribution caused by Earth's rotation. For launches to low-Earth orbit, γ ranges from 0 for a due east launch to 3.0 deg for launch into a polar orbit. The approximation for γ in Eq. (6-42c) is accurate to within 0.1 deg for low-Earth orbits. For launches near the ascending node, β is in the first or fourth quadrant and the plus sign applies in Eq. (6-42a). For launches near the descending node, β is in the second or third quadrant and the minus sign applies in Eq. (6-42a).

Let δ , shown in Fig. 6-12, be the angle in the equatorial plane from the nearest node to the longitude of the launch site. We can determine δ from:

$$\cos \delta = \cos \beta / \sin i \quad (6-43)$$

where δ is positive for direct orbits and negative for retrograde orbits. Finally, the *local sidereal time*, LST , of launch is the angle from the vernal equinox to the longitude of the launch site at the time of launch:

$$\begin{aligned} LST &= \Omega + \delta && \text{(launch at the ascending node)} \\ &= \Omega + 180 \text{ deg} - \delta && \text{(launch at the descending node)} \end{aligned} \quad (6-44)$$

where Ω is the right ascension of the ascending node of the resulting orbit.

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Having calculated the launch azimuth required to achieve the desired orbit, we can now calculate the velocity needed to accelerate the payload from rest at the launch site to the required burnout velocity. To do so, we use topocentric-horizon coordinates with velocity components V_S , V_E , V_Z :

$$V_S = -V_{bo} \cos \phi \cos \beta_b \quad (6-45a)$$

$$V_E = V_{bo} \cos \phi \sin \beta_b - V_L \quad (6-45b)$$

$$V_Z = V_{bo} \sin \phi \quad (6-45c)$$

where V_{bo} is the velocity at burnout (usually equal to the circular orbital velocity at the prescribed altitude), ϕ is the flight path angle at burnout, β_b is the launch azimuth at burnout, and V_L is the velocity of the launch site at a given latitude, L , as given by:

$$V_L = (464.5 \text{ m/s}) \cos L \quad (6-46)$$

Equations (6-45c) do not include losses in the velocity of the launch vehicle because of atmospheric drag and gravity—approximately 1,500 m/s for a typical launch vehicle. Also, in Eq. (6-45c) we assume that the azimuth at launch and the azimuth at burnout are the same. Changes in the latitude and longitude of the launch vehicle during powered flight will introduce small errors into the calculation of the burnout conditions. We can calculate the velocity required at burnout from the energy equation if we know the semimajor axis and radius of burnout of the orbit [Eq. (6-4)].

6.5 Orbit Maintenance

Once in their mission orbits, many satellites need no additional orbital adjustments. On the other hand, mission requirements may demand that we maneuver the satellite to correct the orbital elements when perturbing forces have changed them. Two particular cases of note are satellites with repeating ground tracks and geosynchronous equatorial satellites, placed at an assigned longitude.

Using two-body equations of motion, we can show that a satellite will have a *repeating ground track* if it has exactly an integer number of revolutions per integer number of days. Its period, therefore, must be:

$$P = (m \text{ sidereal days}) / (k \text{ revolutions}) \quad (6-47)$$

where m and k are integers, and 1 sidereal day = 1,436,068.167 min. For example, a satellite orbiting the Earth exactly 16 times per day has a period of 89.75 min and a semimajor axis of 6,640 km.

Next we would modify the period of the satellite to account for the drift in the orbital plane caused by the Earth's oblateness (J_2). We can calculate the change in the right ascension of the ascending node, $\Delta\Omega$, because of J_2 , from the two-body orbital elements. In this case the new period is:

$$P_{\text{NEW}} = P_{\text{TWO BODY}} + \Delta\Omega / \dot{\Omega}_{\text{Earth}} \quad (6-48)$$

Because we base the nodal drift on the two-body orbital elements, we must iterate to find the new orbital period and semimajor axis. Continuing with the previous example, assume a perigee altitude of 120 km and an inclination of 45 deg. In this case, we find that the compensated period is 88.20 min and the new semimajor axis is 6,563 km. Several examples of spacecraft placed in orbits with repeating ground tracks are shown in Table 6-7.

TABLE 6-7. Examples of Repeating Ground Tracks.

Satellite	Inclination (deg)	Semimajor Axis (km)	Revs	Days
SEASAT	108.0	7,169.0	43	3
LANDSAT 4/5	98.2	7,077.7	233	16
GEOSAT	108.05	7,162.7	244	17

The Earth's oblateness also causes the direction of perigee to rotate around the orbit. If the orbit is noncircular and the mission places limits on the altitude over specific targets, we must control the location of perigee. One possibility is to select the inclination of the orbit to be at the critical inclination (63.4 deg for a direct orbit and 116.4 deg for a retrograde orbit), so the location of perigee is fixed. If other constraints make this selection impossible, we must maintain the orbit through orbital maneuvers. We can change the location of perigee by changing the flight-path angle by an angle θ . Only the direction of the velocity vector changes, so we can find the change in velocity from the equation for a simple plane change:

$$\Delta V = 2V \sin \theta/2 \quad (6-49)$$

A final consideration for a low-altitude orbit with repeating ground tracks is the change in the semimajor axis and eccentricity due to atmospheric drag. Drag causes the orbit to become smaller. As the orbit becomes smaller, the period also decreases, causing the ground track to appear to shift eastward. If we specify some tolerance, such as a maximum distance between the actual and desired ground tracks, the satellite must periodically maneuver to maintain the desired orbit.

We can use Eq. (6-22) to calculate the change in semimajor axis per revolution of the orbit. Given the change in the size of the orbit, we can also determine the change in the period:

$$\Delta P = 3\pi \Delta a / (na) \quad (6-50)$$

If constraints exist for either the period or semimajor axis of the orbit, we can use Eqs. (6-22) and (6-50) to keep track of the period and semimajor axis until we need to correct the orbit. We can apply a tangential velocity change at perigee to adjust the semimajor axis when required. Again, we can find the current and needed velocities from the energy Eq. (6-4), because we know the size, and therefore the energy, of the two orbits.

Geosynchronous equatorial orbits also require orbital maintenance maneuvers. Satellites in these orbits drift when perturbations occur from the nonspherical Earth and from the third-body gravitational attraction of the Sun and Moon. Matching the period of a geostationary orbit with the Earth's rotational velocity results in a resonance with the J_2 term in the geopotential. This resonance term results in a transverse acceleration, that is, an acceleration in the plane of the orbit. This causes the satellite to drift in longitude (*East-West drift*). The Sun and the Moon cause out-of-plane accelerations which make the satellite drift in latitude (*North-South drift*).

North-south stationkeeping is necessary when mission requirements limit inclination drift. If not corrected, the inclination of the orbit varies between 0 and 1.5 deg with a period of approximately 55 years. The approximate equations to solve for the worst-case change in velocity are:

$$\Delta V_{MOON} = 102.67 \cos \alpha \sin \alpha \text{ (m/s/yr)} \quad (6-51)$$

$$\approx 36.93 \text{ m/s per year, for } i = 0$$

$$\Delta V_{SUN} = 40.17 \cos \gamma \sin \gamma \text{ (m/s/yr)} \quad (6-52)$$

$$\approx 14.45 \text{ m/s/yr, for } i = 0$$

where α is the angle between the orbital plane and the Moon's orbit, and γ is the angle between the orbital and ecliptic plane.

The transverse acceleration caused by resonance with the J_2 term results in periodic motion about either of two stable longitudes at approximately 75 deg and 255 deg East longitude. If we place a satellite at any other longitude, it will tend to orbit the closest of these two longitudes, resulting in East-West drift of up to 180 deg with periods of up to 900 days. Suppose a mission for a geostationary satellite specifies a required longitude, l_D . The change in velocity required to compensate for the drift and maintain the satellite in the vicinity of the specified longitude is:

$$\Delta V = 1.715 \sin(2|l_D - l_i|) \quad (6-53)$$

where l_D is the desired longitude, l_i is the closest stable longitude, and ΔV is in m/s/yr.

For example, we find the velocity change required for one year if the desired longitude is 60 deg west as

$$l_D = -60 \text{ deg} \quad l_i = 255 \text{ deg}$$

$$\Delta V = 1.715 \text{ m/s/yr}$$

After the mission of the satellite is complete, several options exist, depending on the orbit. We may allow low-altitude orbits to decay and reenter the atmosphere or use a velocity change to speed up the process. We may also boost satellites at all altitudes into benign orbits to reduce the probability of collision with active payloads, especially at synchronous altitudes or when the satellite is part of a large constellation. Because coplanar velocity changes are more efficient than plane changes, we would normally apply tangential changes in velocity. Their magnitude would depend on the difference in energy of the two orbits. Generally, a change in altitude of a few hundred kilometers is sufficient to prevent collisions within a constellation. If we increase the altitude of the orbit for disposal, we must make sure atmospheric drag does not cause the disposal orbit to decay and return the spacecraft to the original altitude of the constellation. If we choose to deorbit our satellite, the velocity change required to deorbit a satellite in a circular orbit at initial altitude, H_i , and velocity, V , is:

$$\Delta V_{deorbit} = V \left[1 - \sqrt{\frac{2(R_E + H_e)}{2R_E + H_e + H_i}} \right] \quad (6-54a)$$

$$= V \left[\frac{H_i - H_e}{4(R_E + H_e)} \right] \quad (6-54b)$$

where R_E is the radius of the Earth and H_e is the perigee altitude at the end of the burn.

It is not necessary to reduce perigee altitude to 0 km. Choosing a 50 km deorbit altitude would reduce the FireSat $\Delta V_{deorbit}$ to 183 m/s. Note that we reduce only

perigee in the deorbit burn. Reducing perigee to 100 to 150 km could result in several orbits over which apogee decreases before the spacecraft reenters and might not allow adequate control of the deorbit conditions.

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Chapter 7

Orbit and Constellation Design

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- 7.1 The Orbit Design Process
- 7.2 Earth Coverage
 - Analytic Approximations; Numerical Simulations; Coverage Figures of Merit*
- 7.3 The ΔV Budget
- 7.4 Selecting Orbits for Earth-Referenced Spacecraft
- 7.5 Selecting Transfer, Parking, and Space-Referenced Orbits
 - Selecting a Transfer Orbit; Parking and Space-Referenced Orbits*
- 7.6 Constellation Design
 - Coverage and Constellation Structure; Summary of Constellation Design*

Chapters 5 and 6 introduced the geometry and physics of spacecraft orbits, as well as formulas for computing orbit parameters. In contrast, this chapter deals with selecting or designing orbits to meet the largest number of mission requirements at the least possible cost. The orbit selection process is complex, involving trades between a number of different parameters. The orbit typically defines the space mission lifetime, cost, environment, viewing geometry, and, frequently, the payload performance. Most commonly, we must trade the velocity required to achieve the orbit as a measure of cost vs. coverage performance.

Chapter 6 lists several references on astrodynamics. Unfortunately, very few of these references contain any discussion of the orbit selection or design process. Chobotov [1996] and Vallado [1997] have some information. By far the most extensive discussion of this topic is in Wertz [2001]. Scoop [1994] contains an excellent discussion of the design of geostationary orbits.

Ordinarily, spacecraft will be in various orbits during the space mission life. These could include, for example, a *parking orbit* for spacecraft checkout or storage, a *transfer orbit* to move it from the *injection orbit* where the spacecraft separated from the launch vehicle to its *operational orbit* or orbits for mission activities, and possibly a final *disposal orbit* where the spacecraft will do minimum damage when we are through using it. In early mission design, we pay most attention to defining the spacecraft's operational orbit. However, preliminary mission design needs to consider all mission phases to meet the needs of more complex missions.